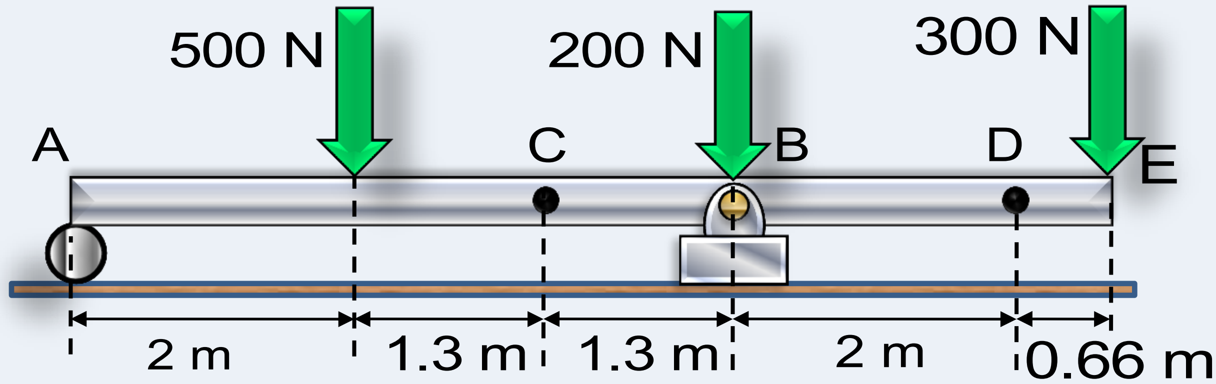
EQUILIBRIUM OF RIGID BODY

I hear and I forget,
I see and I remember,
I do and I understand
Chinese Proverb

Background Music: Chiquitita

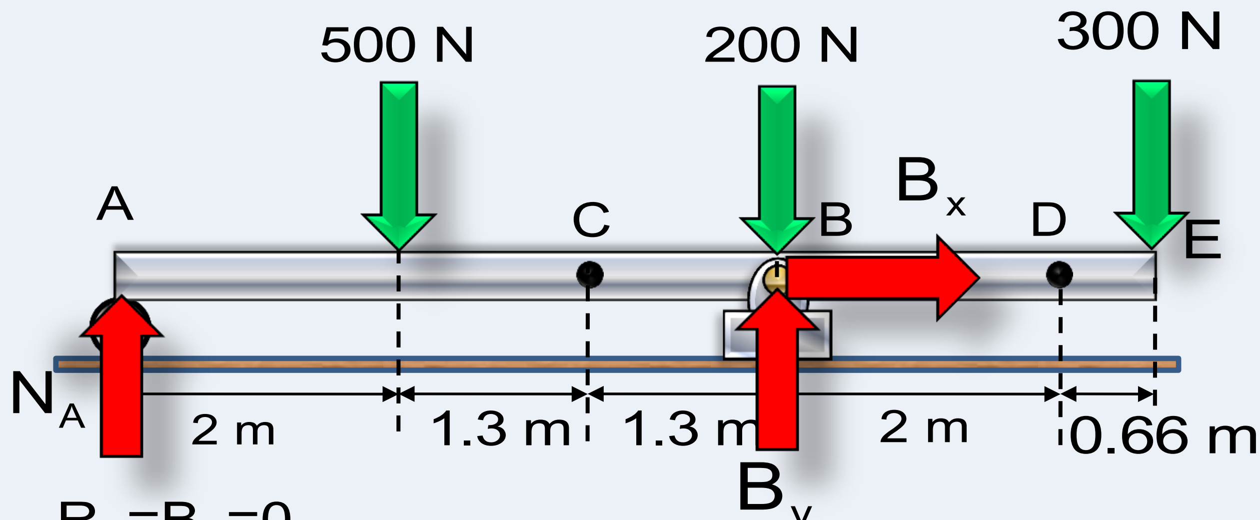
EXAMPLE-6

Determine the normal (axial) force , shear force, and moment at a section passing through points C and D in the beam



SOLUTION

Step One: reactions at the supports



$R_x = B_x = 0$

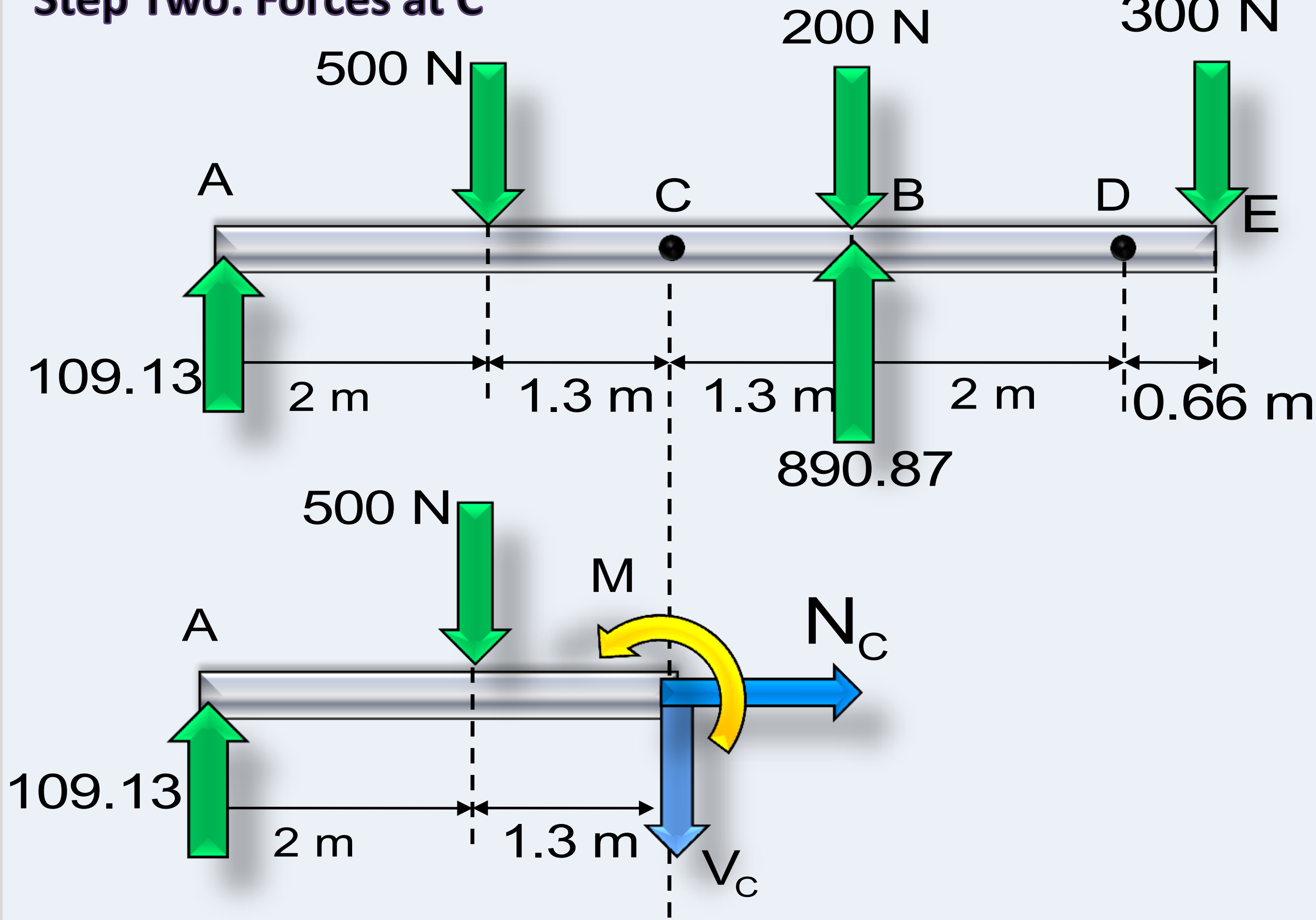
$R_y = N_A - 500 - 200 + B_y - 300 = 0$

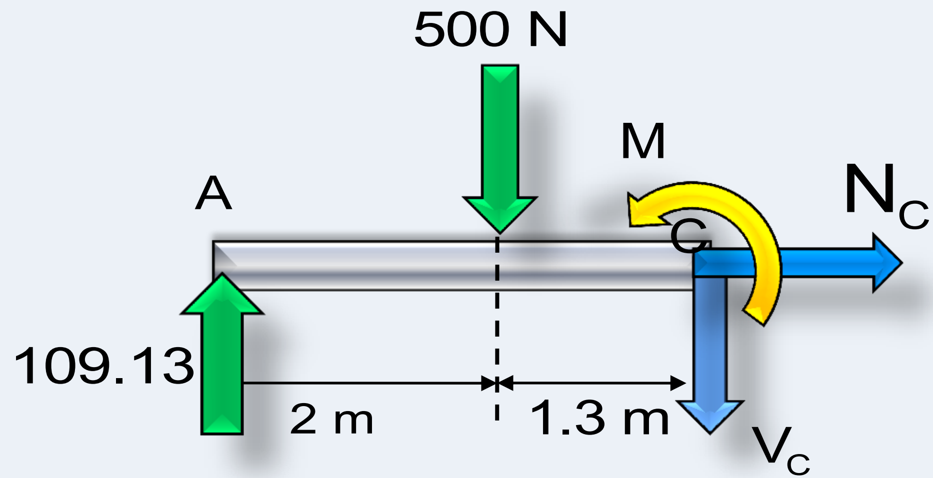
$M_B = -N_A(4.6) + 500(2.6) - 300(2.66) = 0$

Solving the above three equations yields

$N_A = 109.13$ N, $B_y = 890.87$ N

Step Two: Forces at C





$R_x = N_C = 0$

$R_y = 109.13 - 500 - V_C = 0 \rightarrow V_C = 390.87$ N

$M_C = -109.13(3.3) + 500(1.3) + M = 0 \rightarrow M = 289.87$ N-m

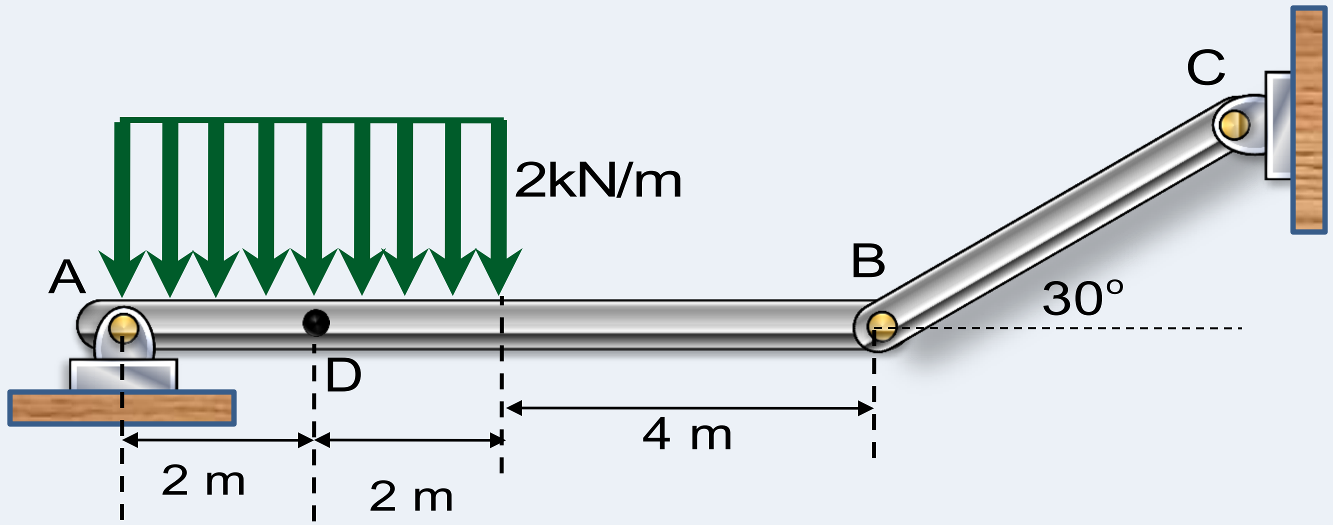
Left to the student:

Obtain the internal forces at point C, by considering the right section of the beam. i.e. (CBED)

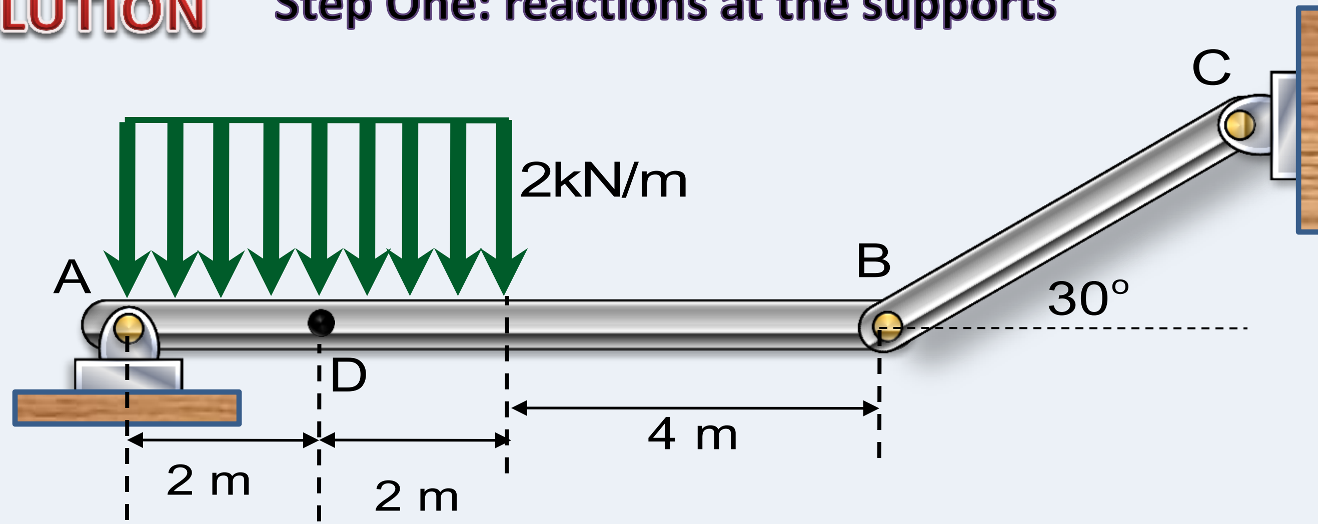
Following the same procedure as above, calculate the internal forces at point D

EXAMPLE-7

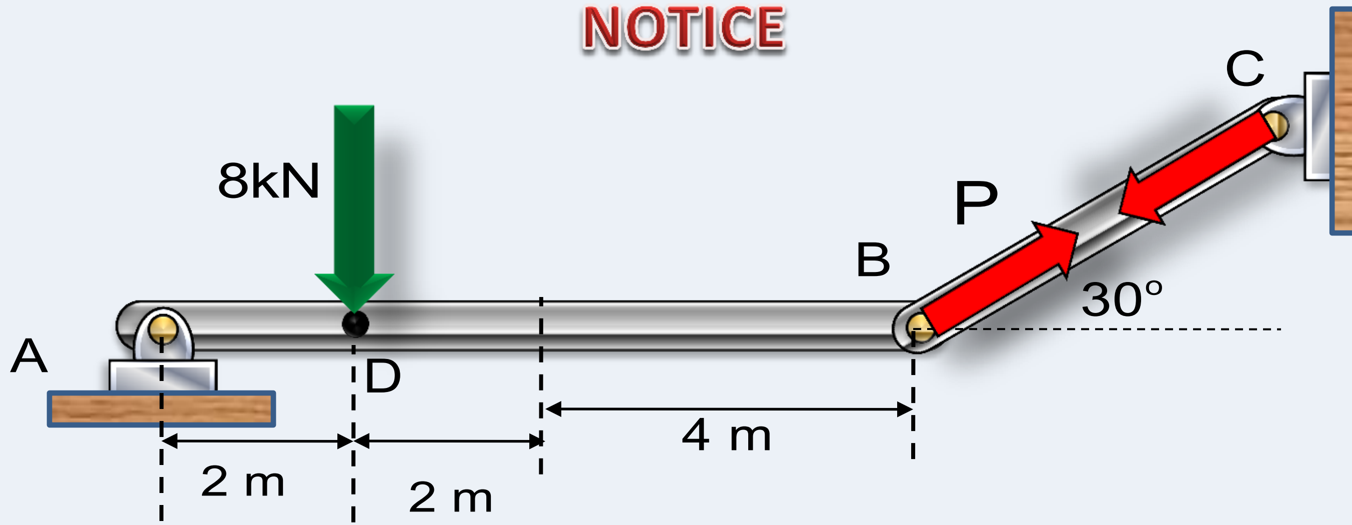
Find the axial force, shear force, and bending moment acting internally on the beam at D.

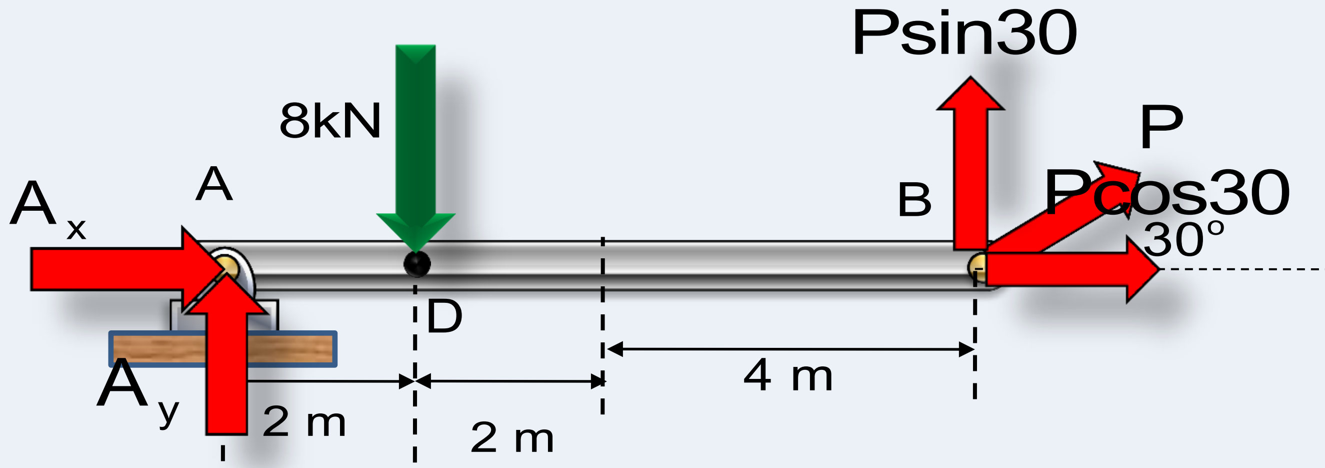


SOLUTION Step One: reactions at the supports



NOTICE





$$R_x = P\cos 30 - A_x = 0$$

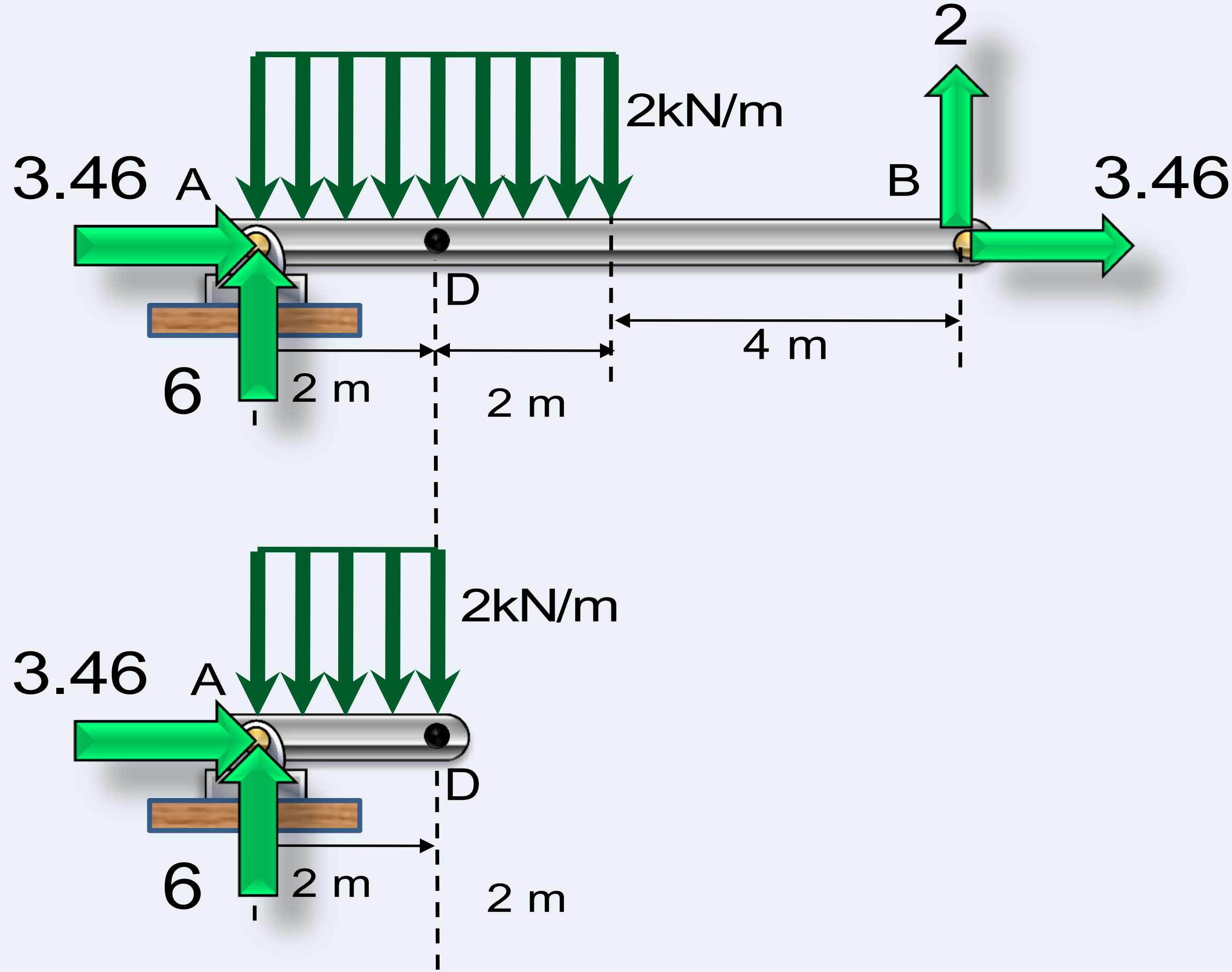
$$R_y = P\sin 30 + A_y - 8 = 0$$

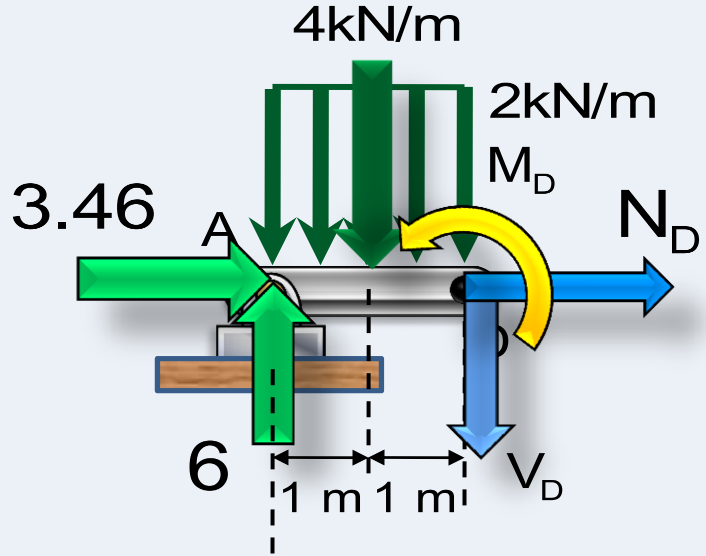
$$M_A = P\sin 30(8) - 8(2) = 0$$

Solving the above equations yields

$P = 4$ kN, $A_x = 3.46$ kN, and , $A_y = 6$ kN

Step Two: internal forces at D





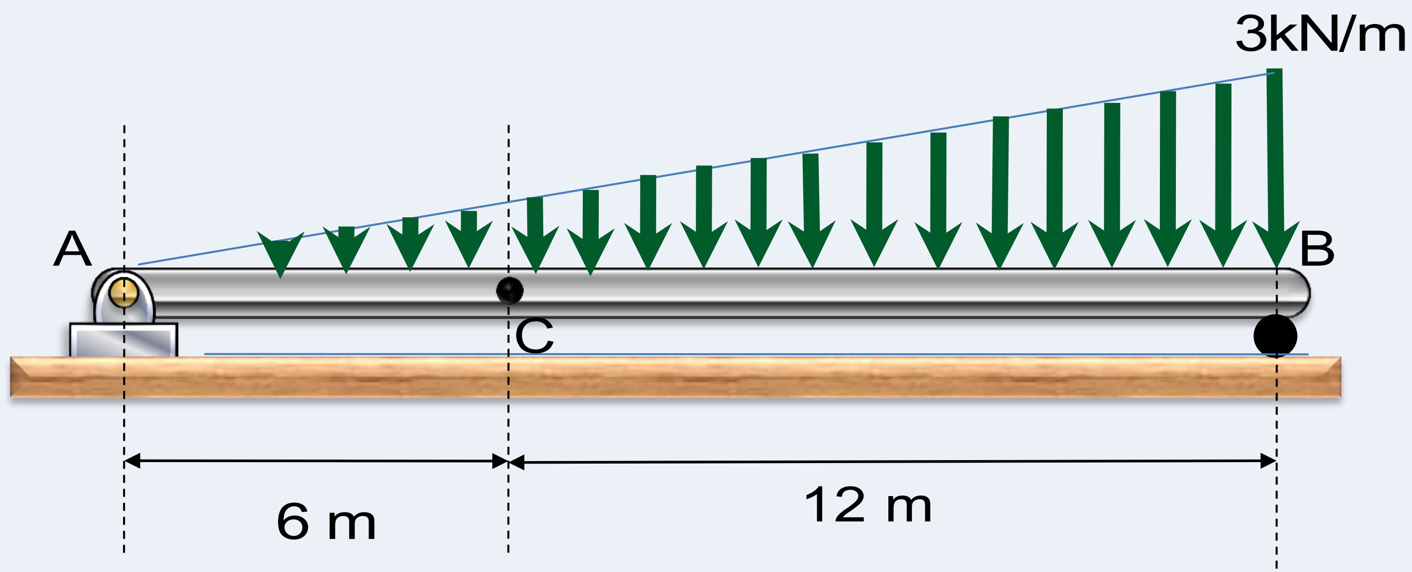
$$R_x = N_D - 3.46 = 0 \Rightarrow N_D = 3.46 \text{ kN}$$

$$R_y = 6 - 4 - V = 0 \Rightarrow V = 2 \text{ kN}$$

$$M_D = -6(2) + 4(1) + M_D = 0 \Rightarrow M_D = 8 \text{ kN.m}$$

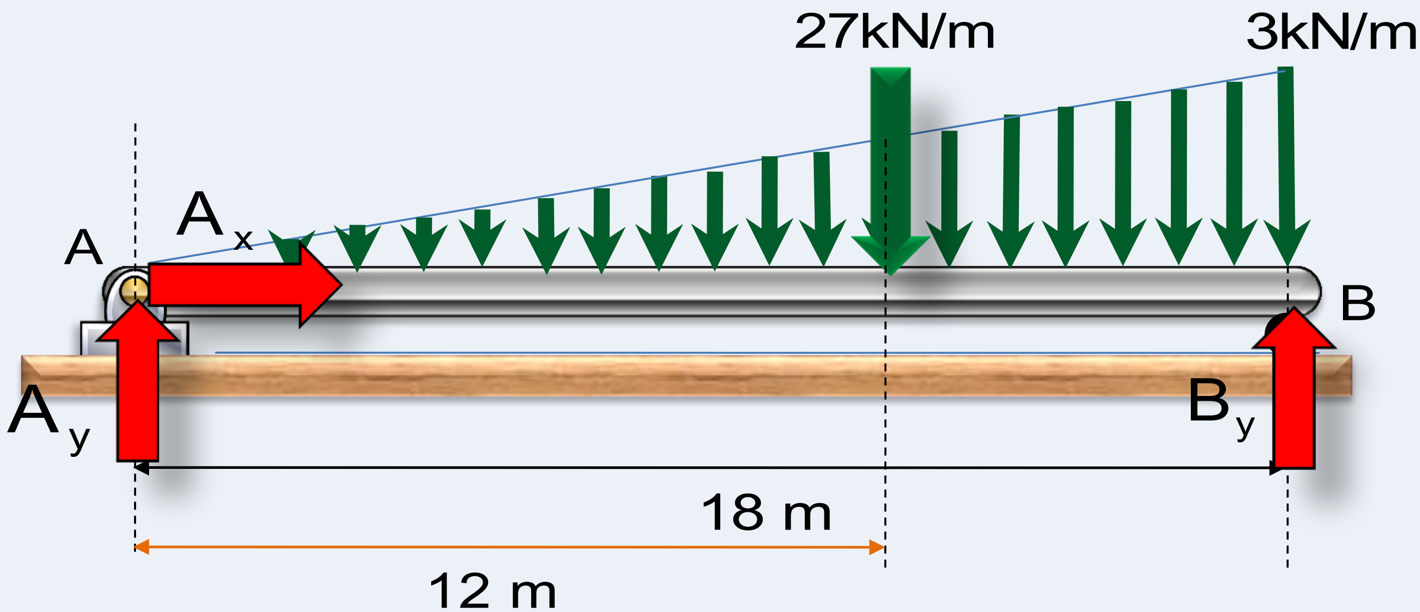
EXAMPLE-8

DETERMINE THE NORMAL FORCE, SHEAR FORCE, AND MOMENT AT A SECTION PASSING THROUGH POINT C IN THE BEAM



SOLUTION

Step One: reactions at the supports

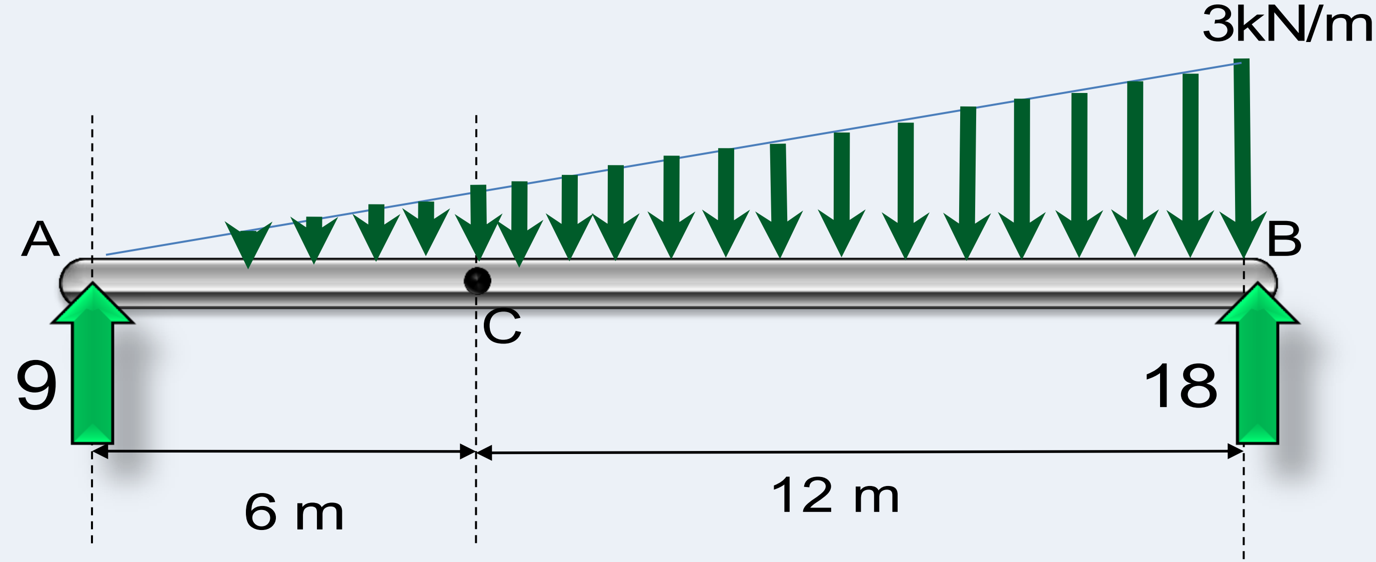


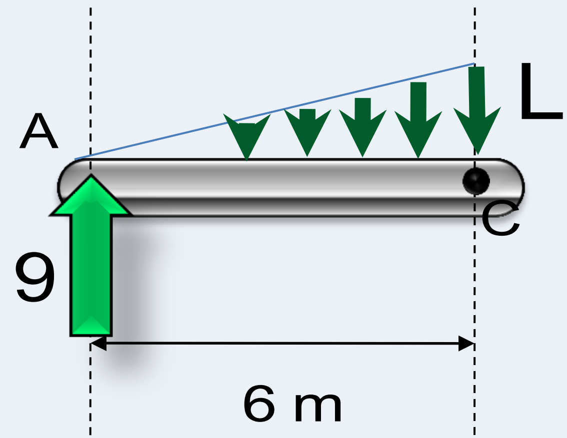
$$R_x = A_x = 0$$
$$R_y = A_y + B_y - 27 = 0$$
$$M_A = -27(12) + B_y(18) = 0$$

Solving the above equations yields

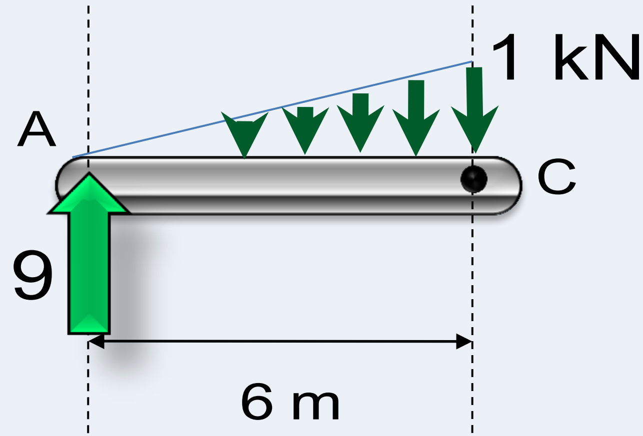
$$A_x = 0, \quad A_y = 9, \quad B_y = 18$$

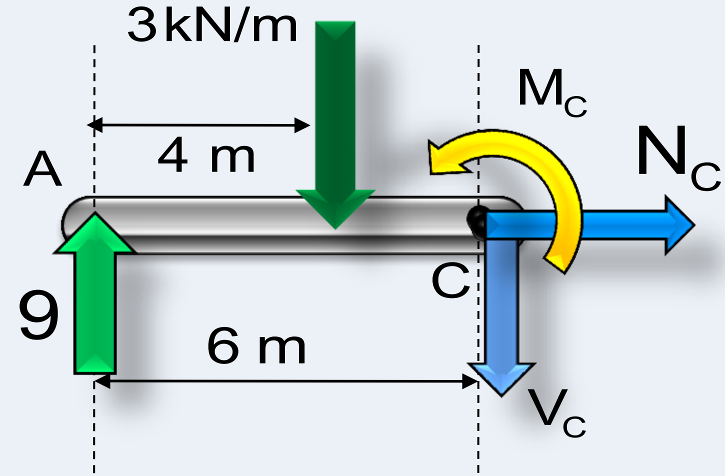
Step Two: internal forces at C





$$\frac{18}{6} = \frac{3}{L} \Rightarrow L = 1\ kN/m$$





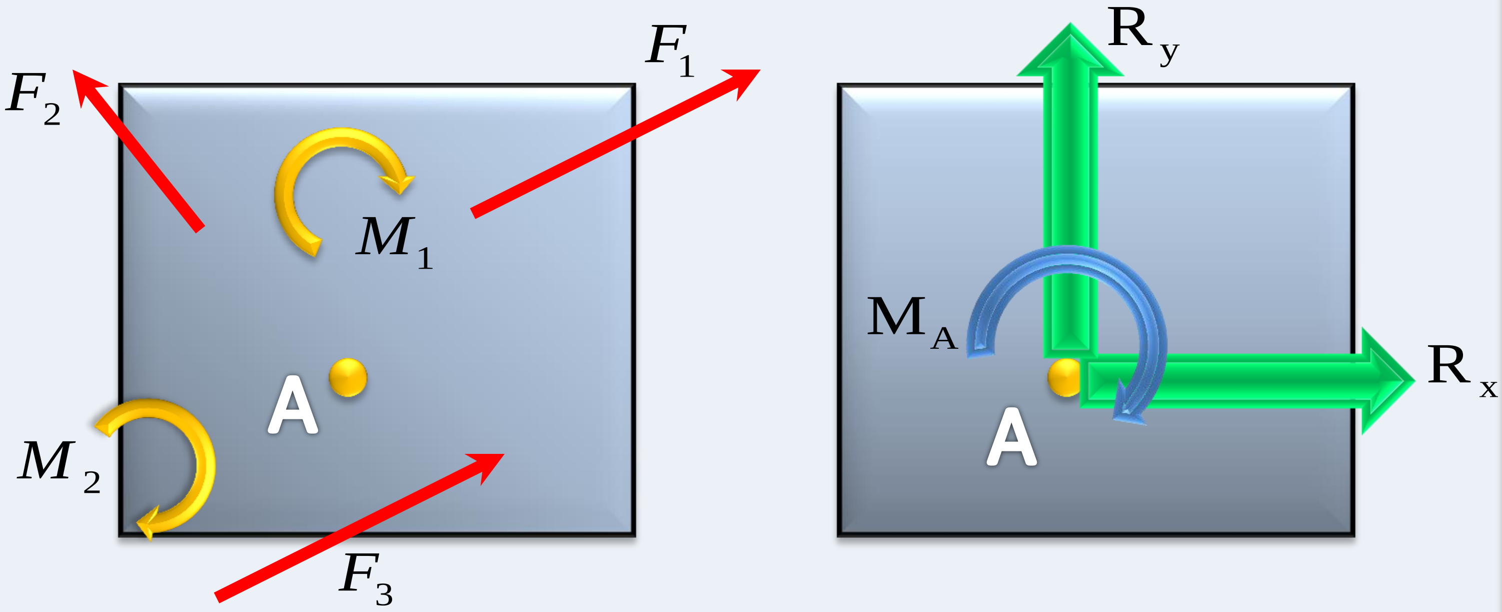
$$R_x = N_C = 0$$

$$R_y = 9 - 3 - V_C = 0 \quad \Rightarrow V_C = 6 \ \text{kN}$$

$$M_A = -3(4) - V_C(6) + M_C = 0 \quad \Rightarrow M_C = 48 \ \text{kN-m}$$

STATIC EQUILIBRIUM (2-D)

EQUILIBRIUM CONDITIONS



STATIC EQUILIBRIUM REQUIRES THAT

$$R_x = F_{x1} + F_{x2} + \ldots = 0$$

$$R_y = F_{y1} + F_{y2} + \ldots = 0$$

$$M_A = \left(\left| \vec{r}_{A/1} x \vec{F}_1 \right| + \left| \vec{r}_{A/2} x \vec{F}_2 \right| + \ldots \right) + \left(M_1 + M_2 + \ldots \right) = 0$$

PROCEDURE FOR PROBLEM SOLVING

The student should know that problem solving is not an art, but rather a professional decision made by recognizing the proper keys. In what follows we present a logical way of thinking to identify the keys that will assist the student in developing his own analytical abilities when solving problems related to equilibrium

1- Determine carefully what data are given and what results are required

2- Select the body (or bodies) to be isolated in a free body diagram (FBD) by examining where the forces of interest are applied. Draw a simple sketch of this body. Label any significant points if they are not already labeled. Choose an xy coordinate system for the description of the components of the force and position vectors. Show this coordinate system in the sketch of step 1

3- Complete the FBD begun in step 2 by showing all forces (LOADS and REACTIONS) acting on the body and their dimensions

4- Choose a convenient point along the line of action of some unknown constraint force and compute the sum of the moments of the forces about this point. Equate the components of the moment sum to zero

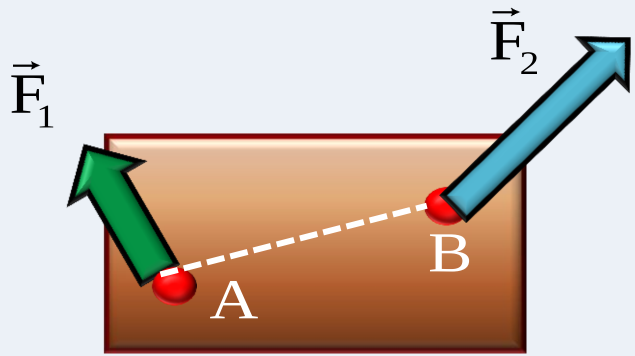
5- Equate to zero each component of the sum of the forces appearing in the FBD.

6- Count the number of scalar equations and the number of unknowns contained in the equations of step 4 and 5. If there are more unknowns than equations, look for another free diagram in the original system that will give information about one or more of the unknowns, Now repeat steps 3,4,5, and 6. If the number of unknowns is equal to the number of equations, Solve the simultaneous equations

TWO-FORCE MEMBERS

TWO FORCE MEMBER

a rigid body on which two forces are acting is called a two force member

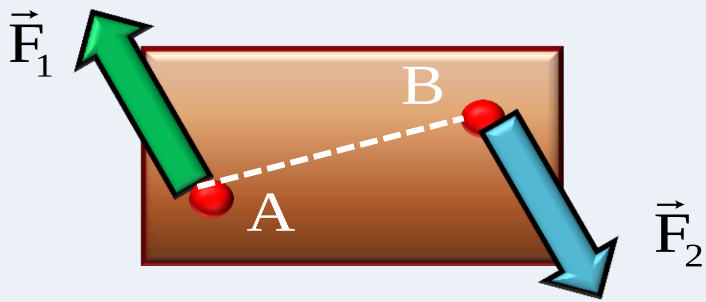


If the body is to be in equilibrium, two requirements must be satisfied

FIRST REQUIREMENT

$$\vec{R} = \vec{R}_x + \vec{R}_y = \vec{F}_1 + \vec{F}_2 = 0 \quad \Rightarrow \quad \vec{F}_1 = -\vec{F}_2$$

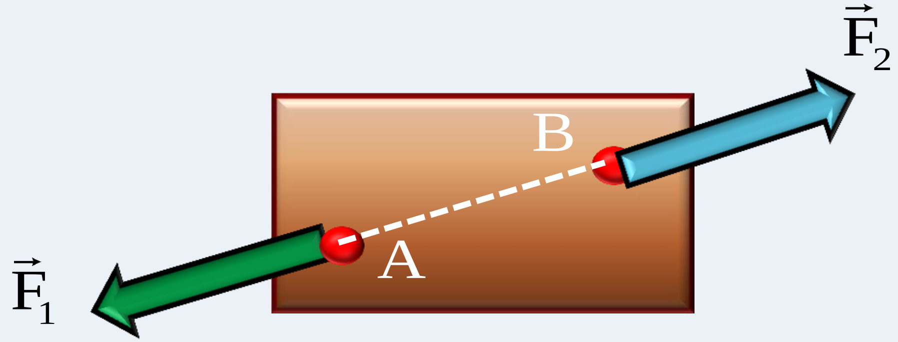
<u>Indicating that the forces must be equal in magnitude, direction, and opposite in sense.</u>



SECOND REQUIREMENT

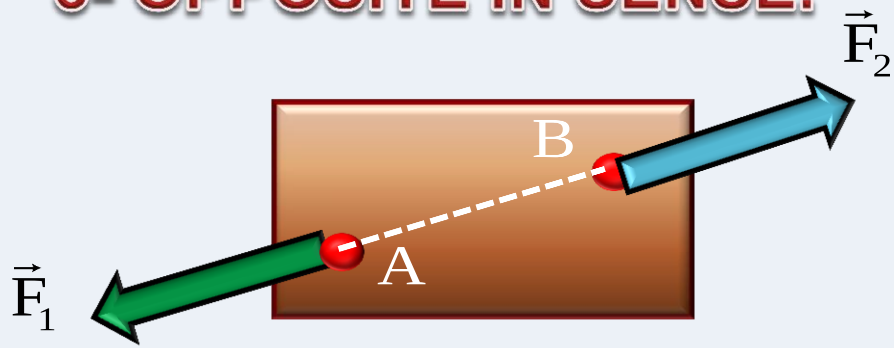
$$M_A = 0$$

Indicating that the forces must be collinear so as not to form a non zero couple.



In conclusion, a two dimensional body subjected to two forces may be in equilibrium only when the forces are

1- EQUAL IN MAGNITUDE,
2- COLLINEAR, AND
3- OPPOSITE IN SENSE.

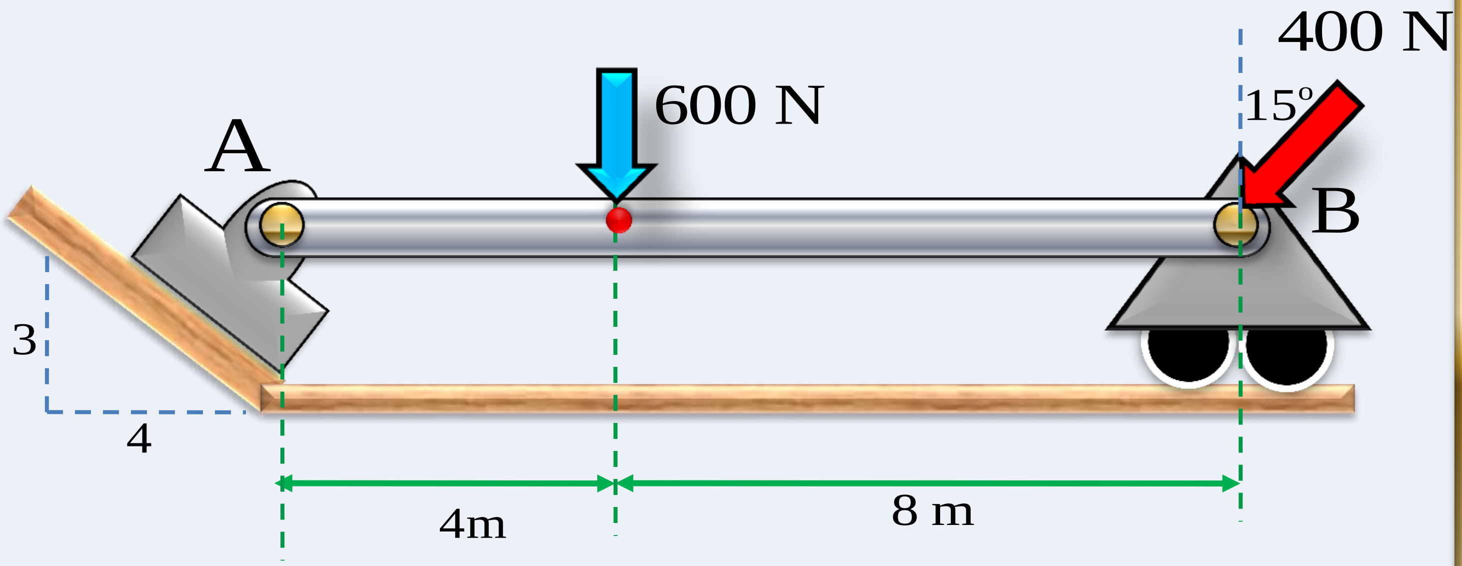


EXAMPLE

The beam AB has pin and roller supports and is subjected to loads 600N and 400 N as shown. The following is required;

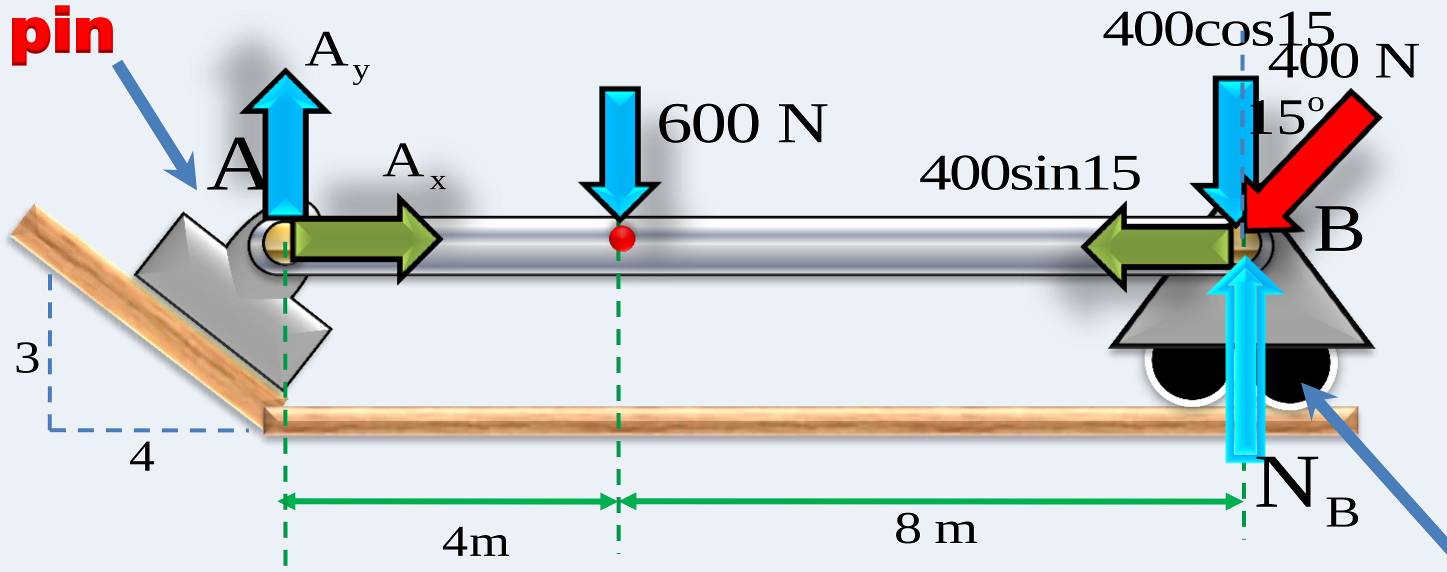
A- draw the free-body diagram of beam AB

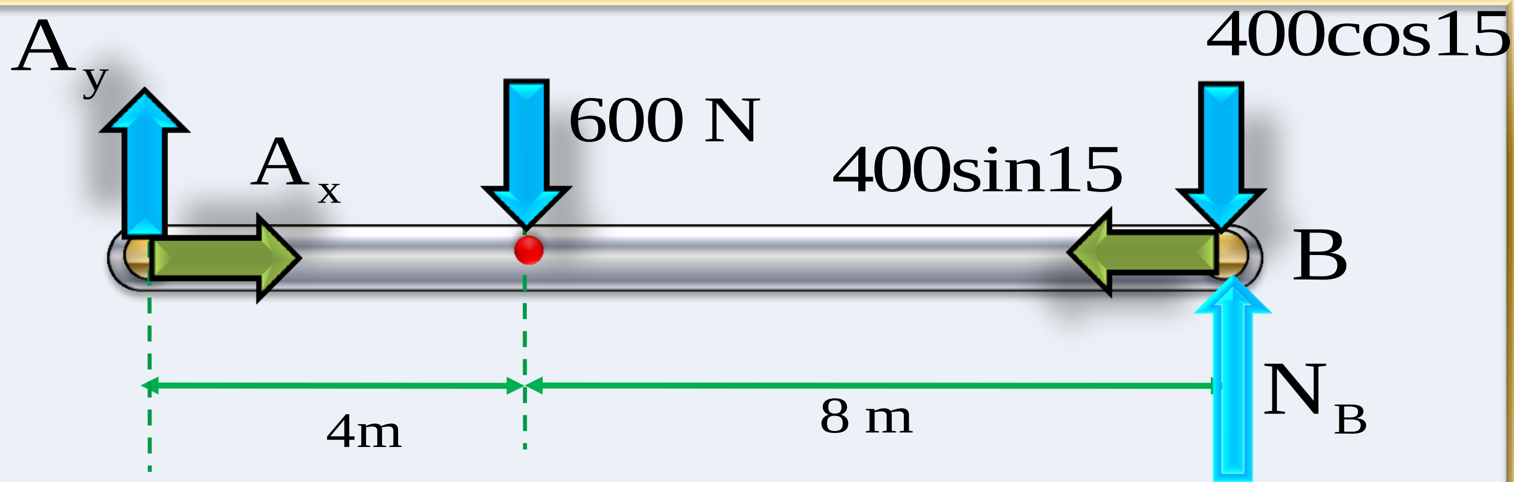
B- determine the reactions at the supports.



SOLUTION

Free Body Diagram





Equilibrium equations

$$R_x = A_x - 400\sin 15 = 0$$

$$R_y = A_y + N_B - 400\cos 15 - 600 = 0$$

$$M_B = -A_y(12) + 600(8) = 0$$

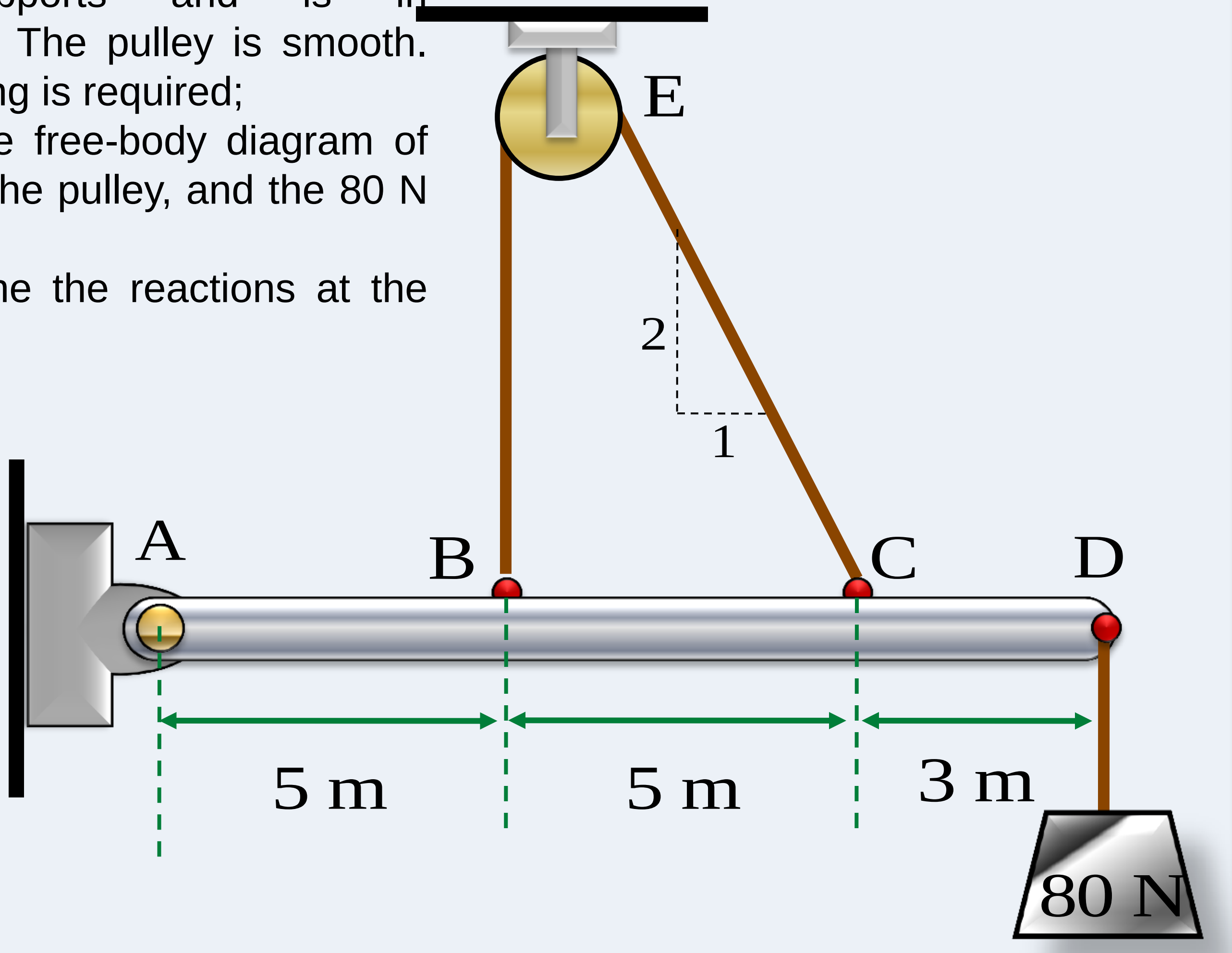
Solve and get three reactions

EXAMPLE

The beam ABCD has pin and cable supports and is in equilibrium. The pulley is smooth. The following is required;

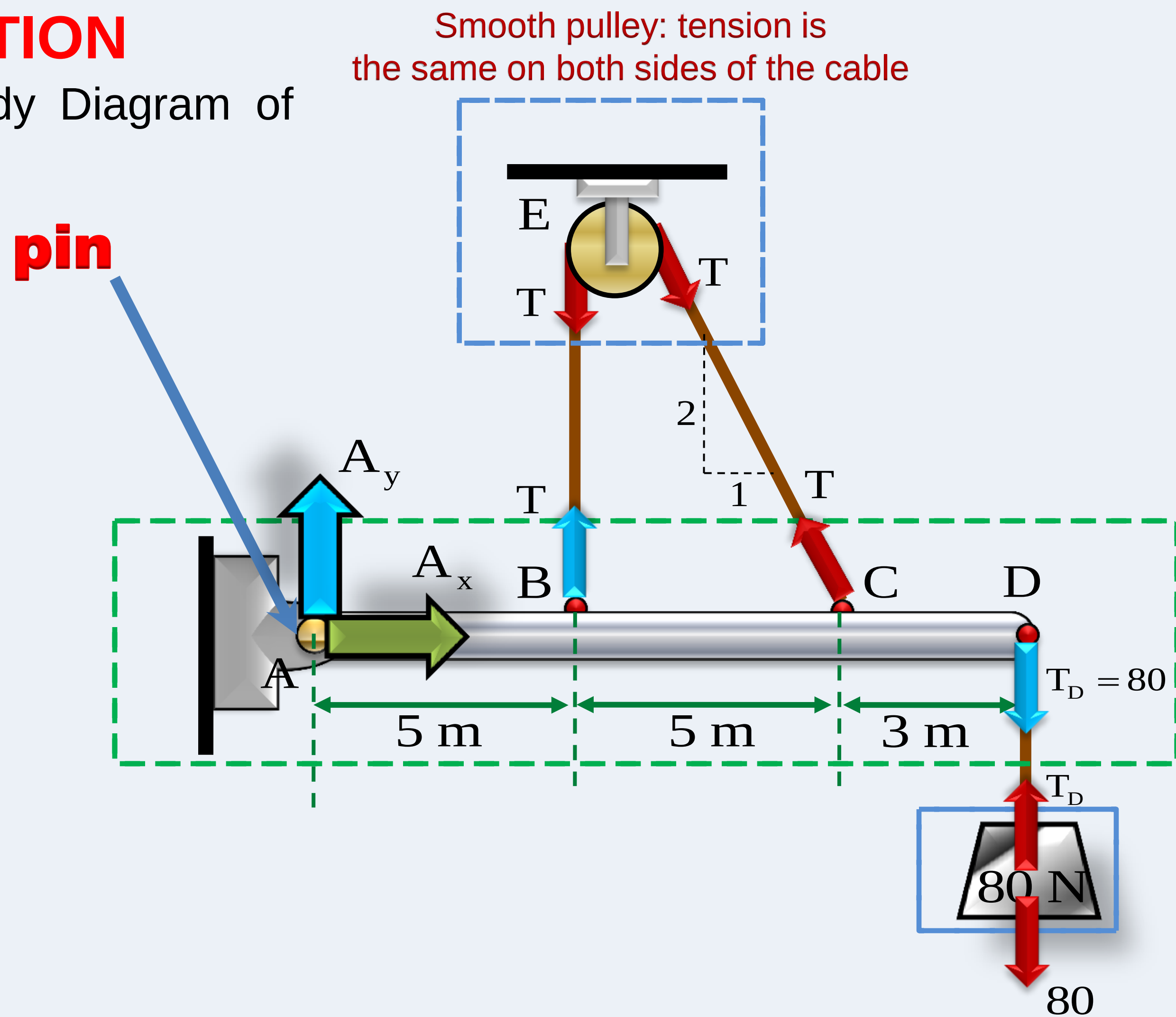
A- draw the free-body diagram of the beam, the pulley, and the 80 N weight

B- determine the reactions at the supports.

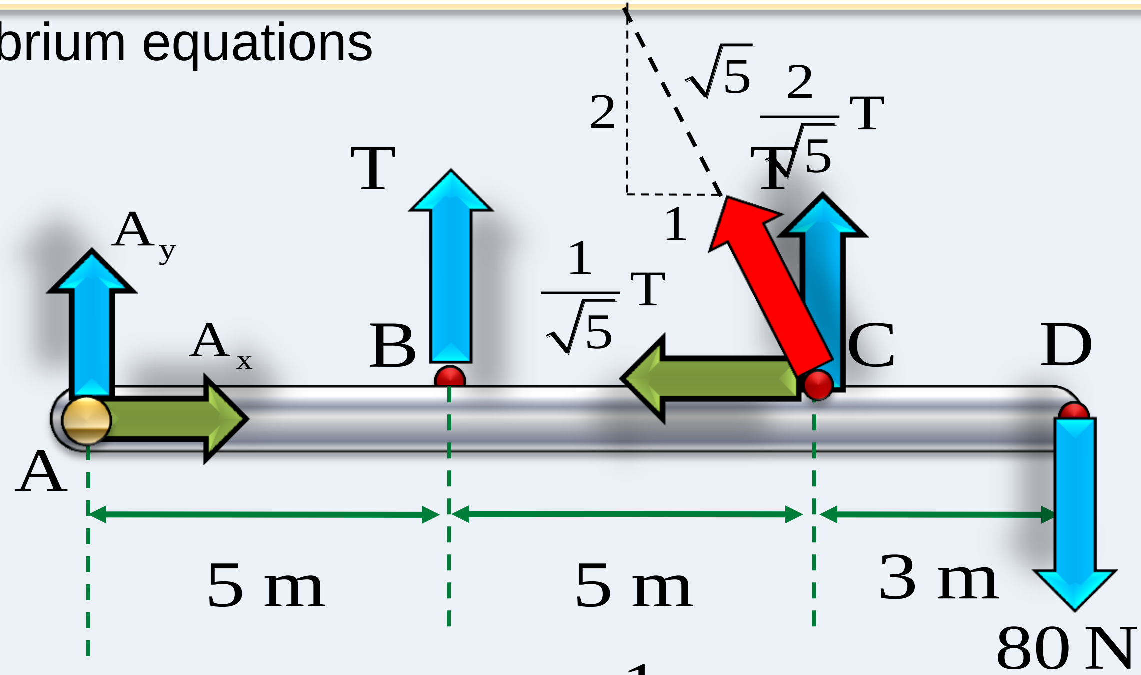


SOLUTION

Free Body Diagram of bar AD



Equilibrium equations



$$R_x = A_x - \frac{1}{\sqrt{5}}T = 0$$

$$R_y = A_y + T + \frac{2}{\sqrt{5}}T - 80 = 0$$

$$M_A = T(5) + \frac{2}{\sqrt{5}}T(10) - 80(13) = 0$$

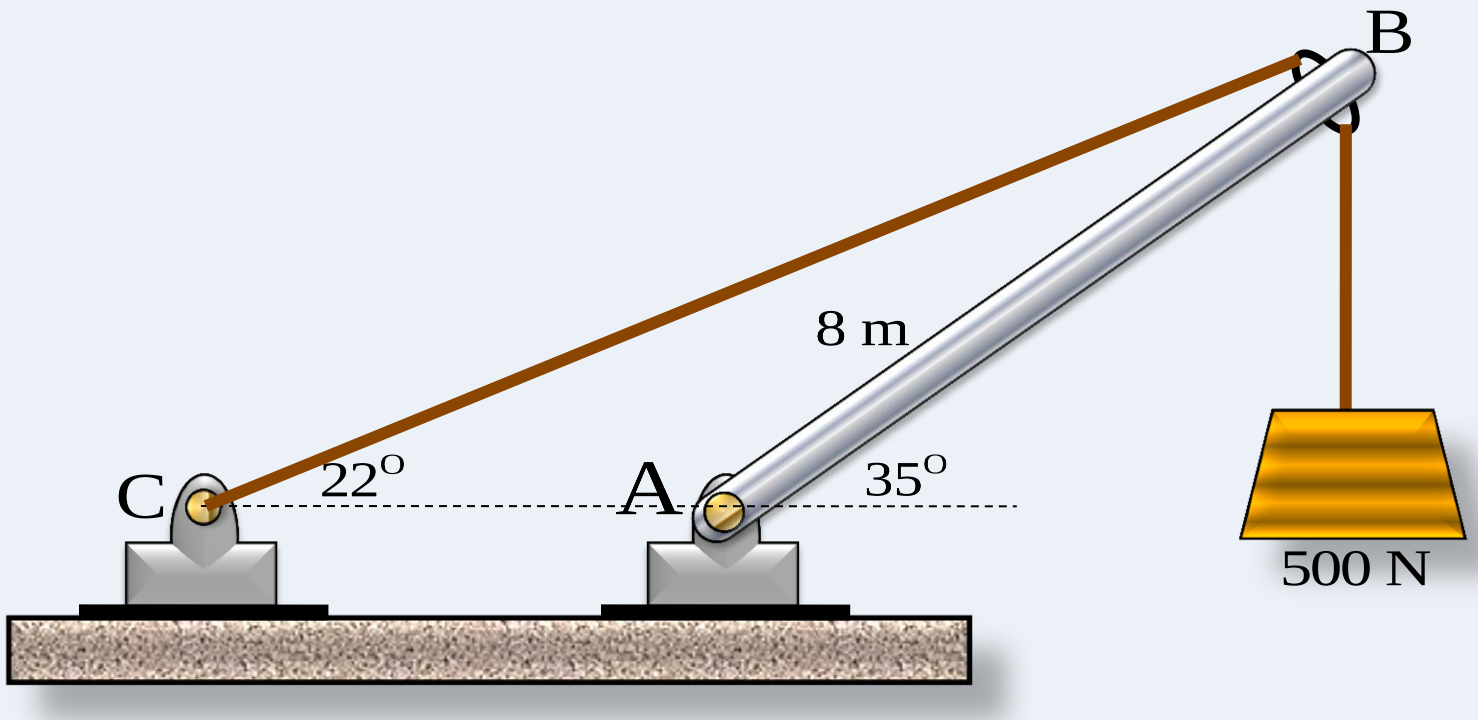
Three equations & three unknowns, solve and get results

EXAMPLE

The beam AB has pin and cable supports and is in equilibrium.
The following is required;

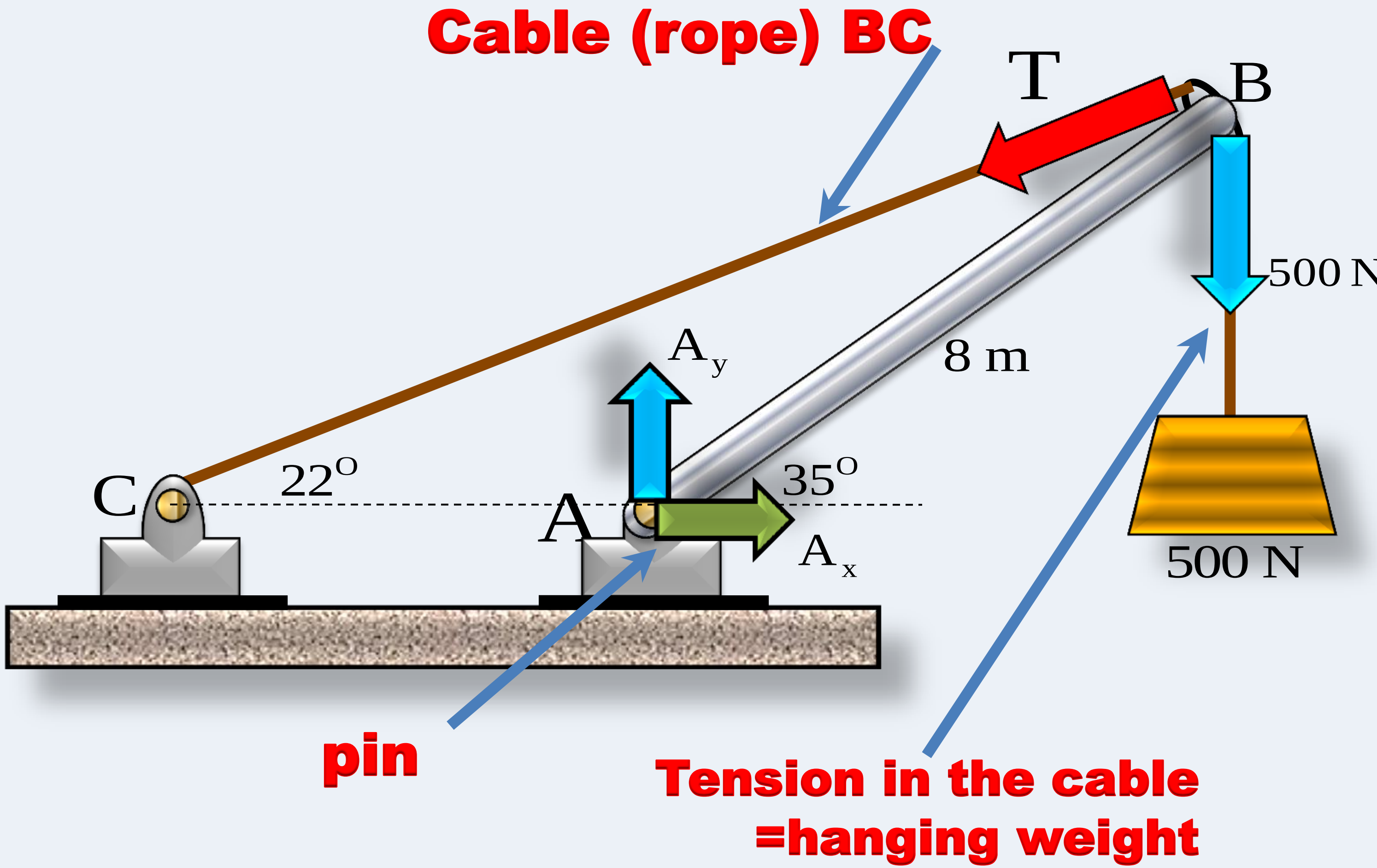
A- draw the free-body diagram of beam AB

B- determine the reactions at the supports.

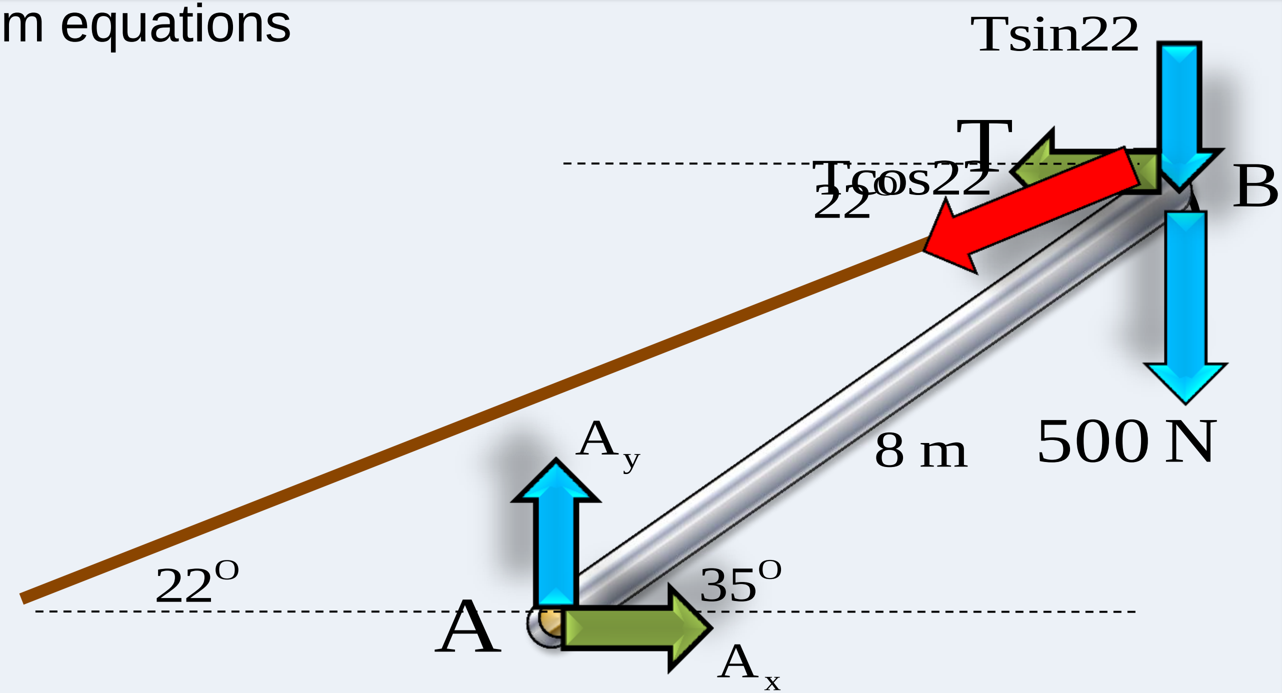


SOLUTION

Free Body Diagram



Equilibrium equations



$$R_x = A_x - T\cos 22 = 0$$

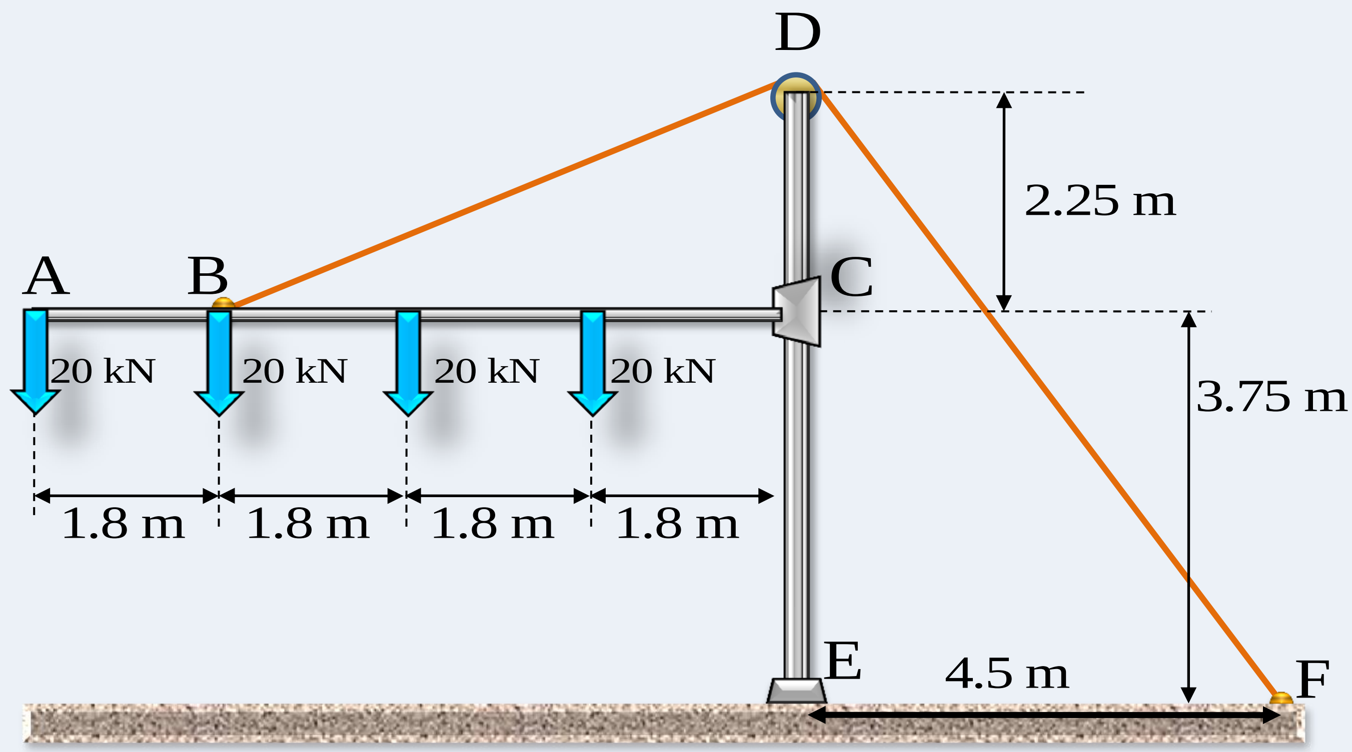
$$R_y = A_y - T\sin 22 - 500 = 0$$

$$M_A = -\left(T\sin 22 + 500\right)(8\cos 35) + T\cos 22\,(8\sin 35) = 0$$

Three equations & three unknowns, solve and get results

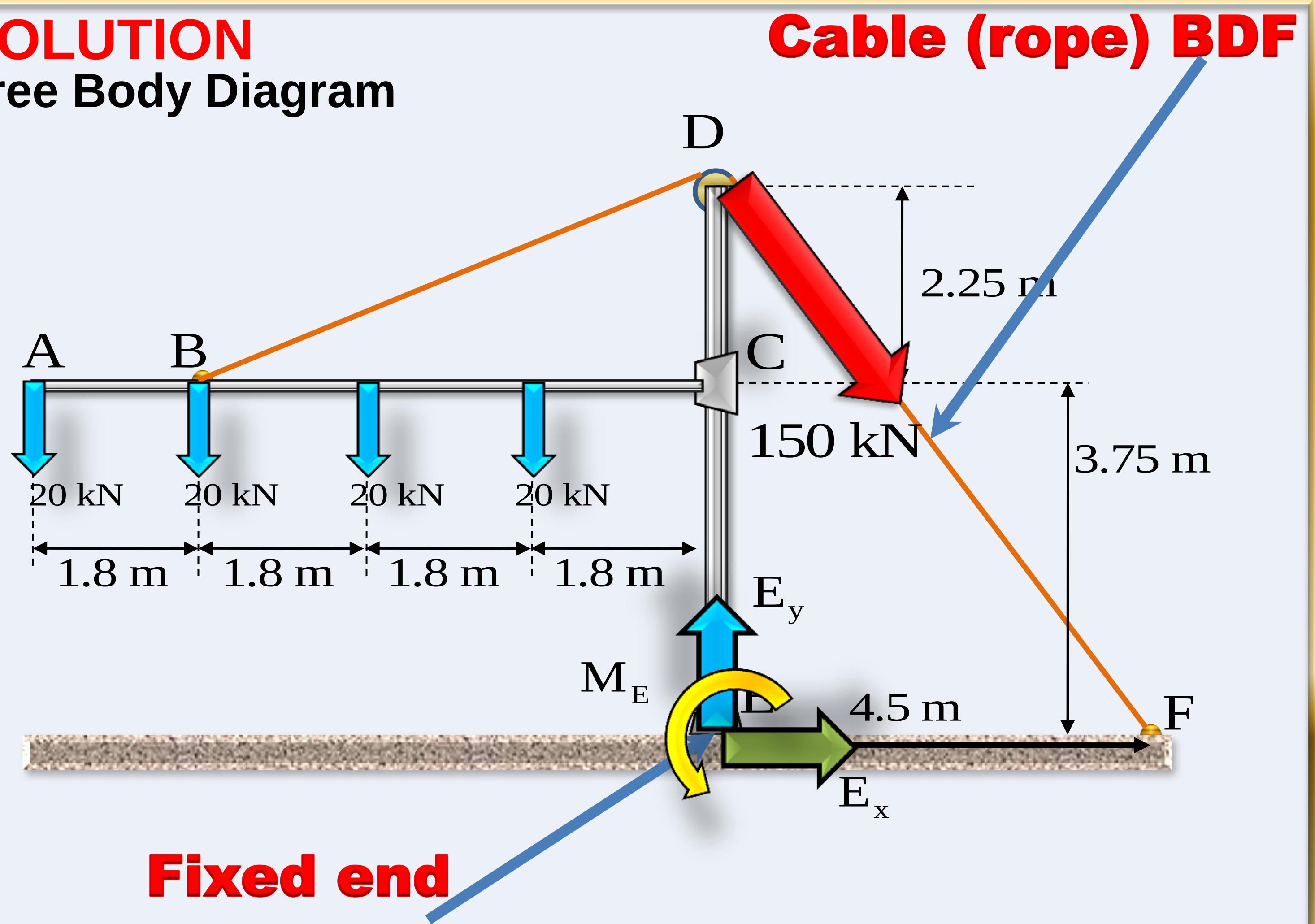
EXAMPLE

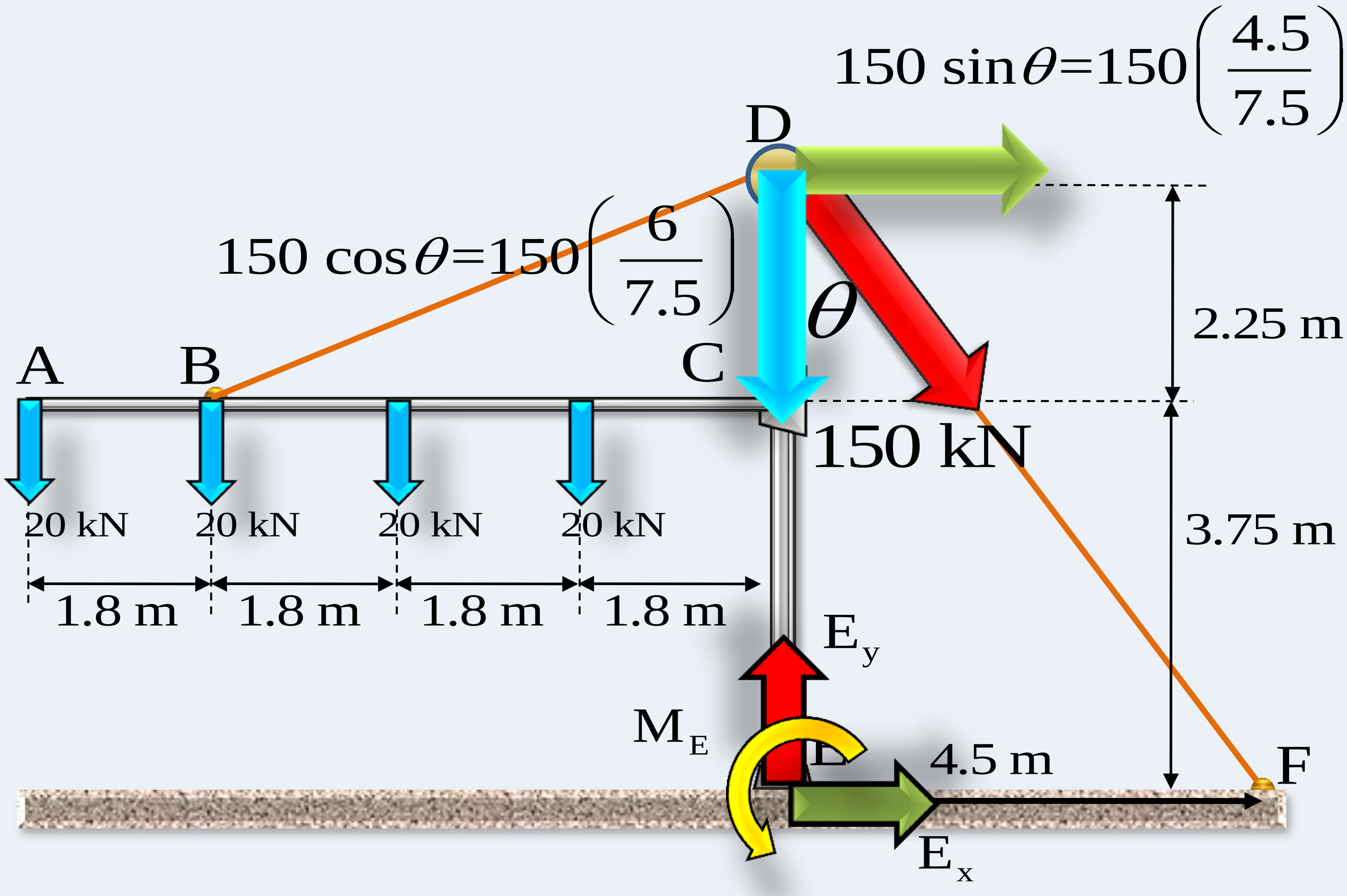
The frame shown supports part of the roof of a small building. Knowing that the tension in the cable is 150 N, determine the reaction at the fixed end E



SOLUTION

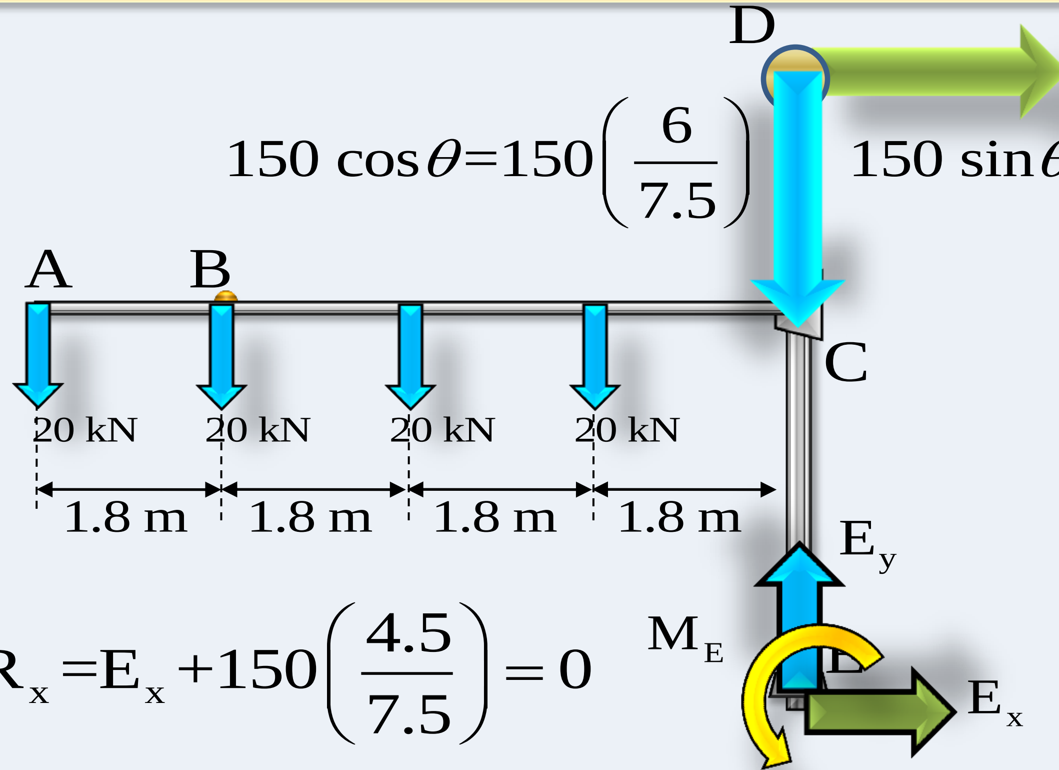
Free Body Diagram





150 sin θ=150 (4.5/7.5)
D
150 cos θ=150 (6/7.5)
θ
2.25 m
A
B
C
150 kN
20 kN
20 kN
20 kN
20 kN
3.75 m
1.8 m
1.8 m
1.8 m
1.8 m
E_y
M_E
E
4.5 m
F
E_x

$$150\cos\theta = 150\left(\frac{6}{7.5}\right) \qquad 150\sin\theta = 150\left(\frac{4.5}{7.5}\right)$$



$$R_x = E_x + 150\left(\frac{4.5}{7.5}\right) = 0$$

$$R_y = -80 - 150\left(\frac{6}{7.5}\right) + E_y = 0$$

$$M_E = -150\left(\frac{4.5}{7.5}\right)(6) + M_y + 20(1.8 + 3.6 + 5.4 + 7.2) = 0$$

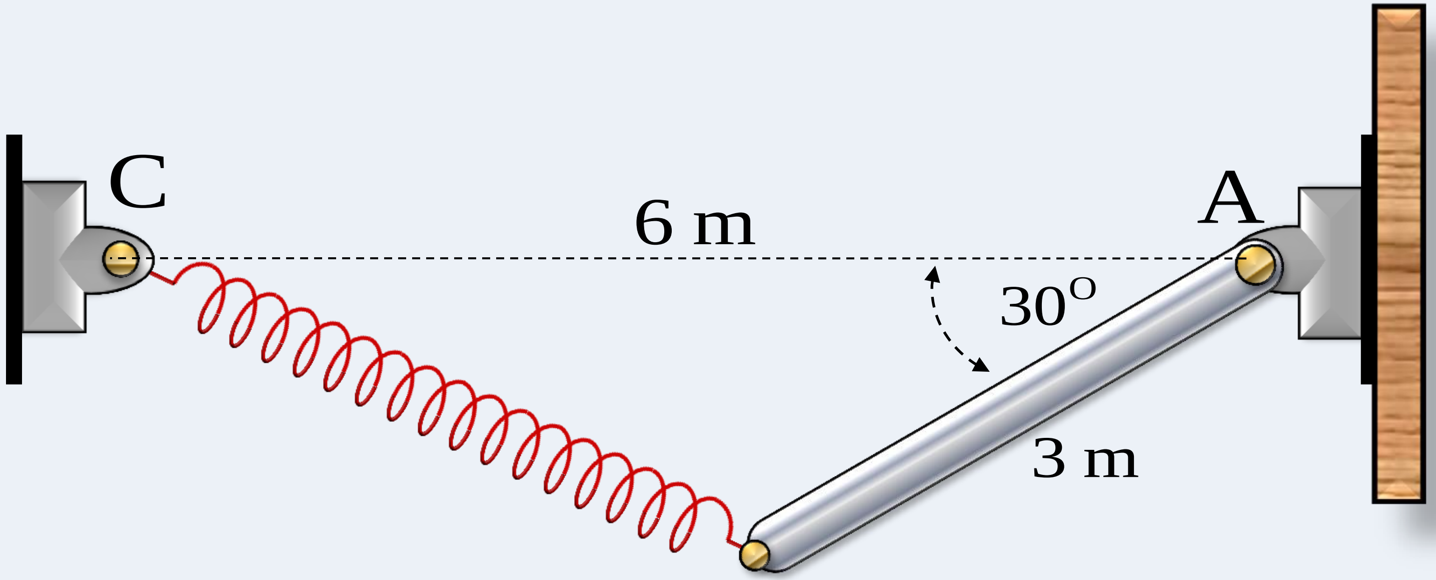
Three equations & three unknowns, solve and get results

EXAMPLE

The uniform bar AB of mass 10 kg has pin and spring supports and is in equilibrium. The unstretched length of the spring is 3 m. The following is required;

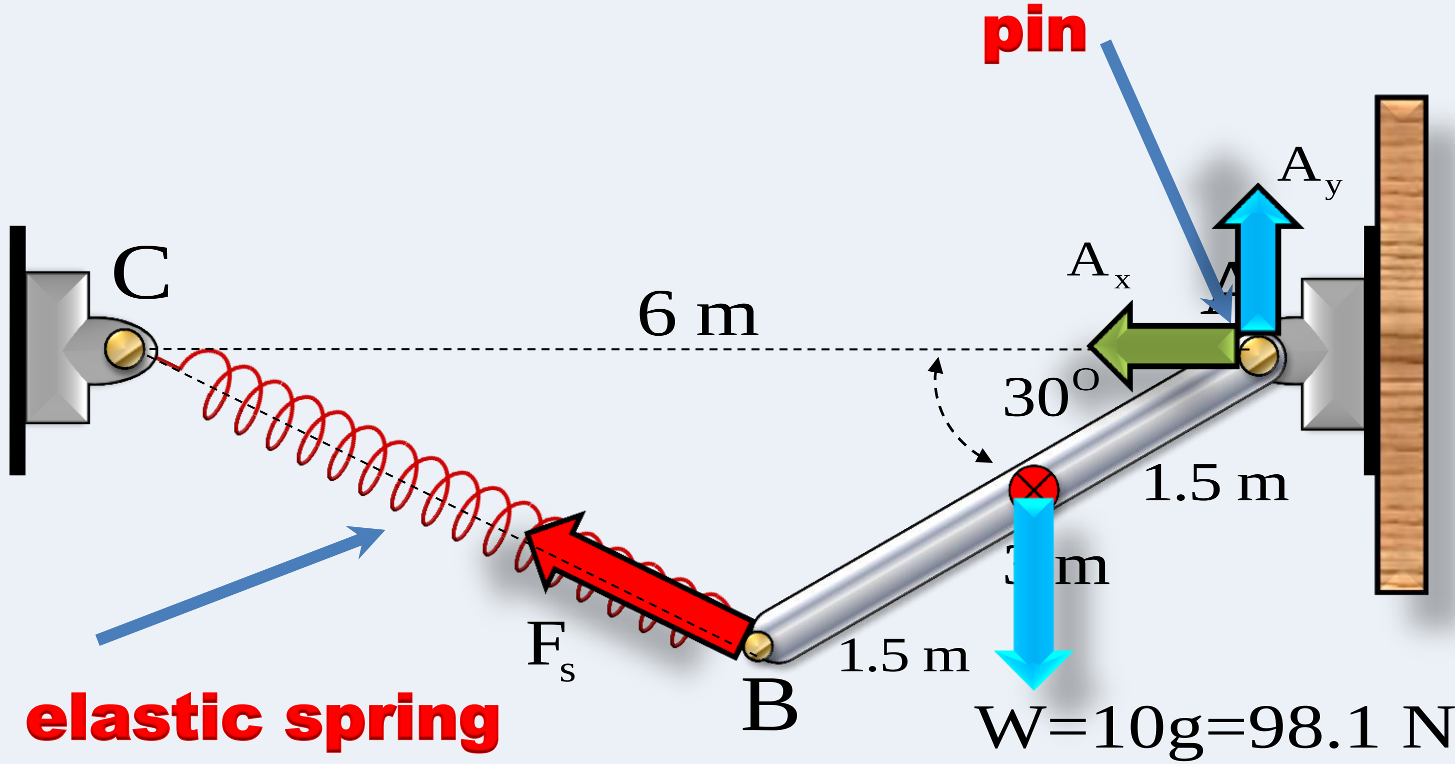
A- draw the free-body diagram of the bar

B- determine the reactions at the supports and the springs constant.

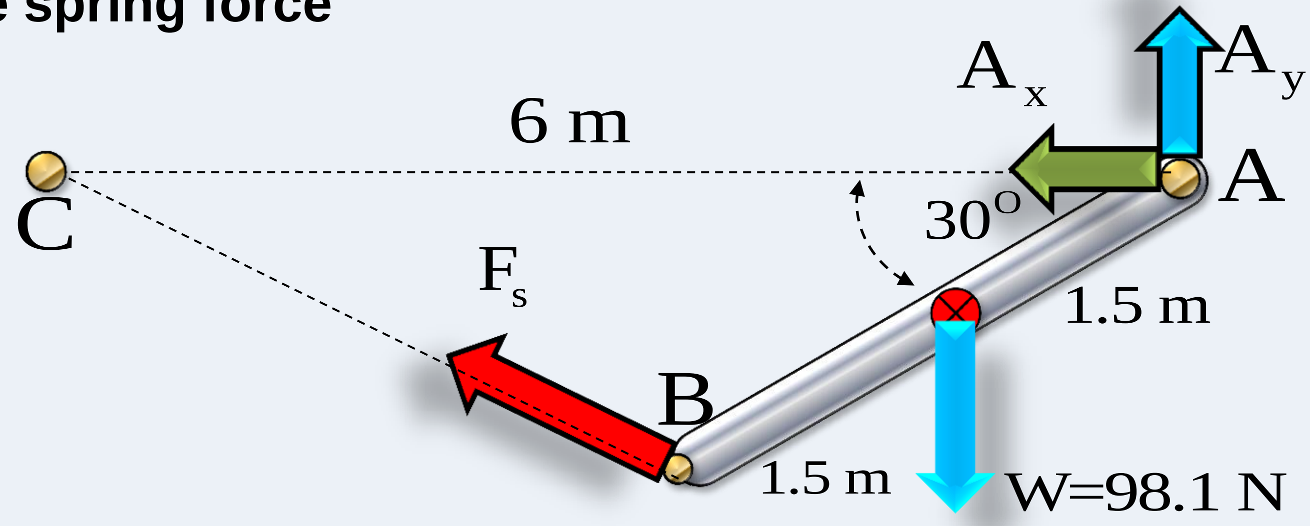


SOLUTION

Free Body Diagram



The spring force



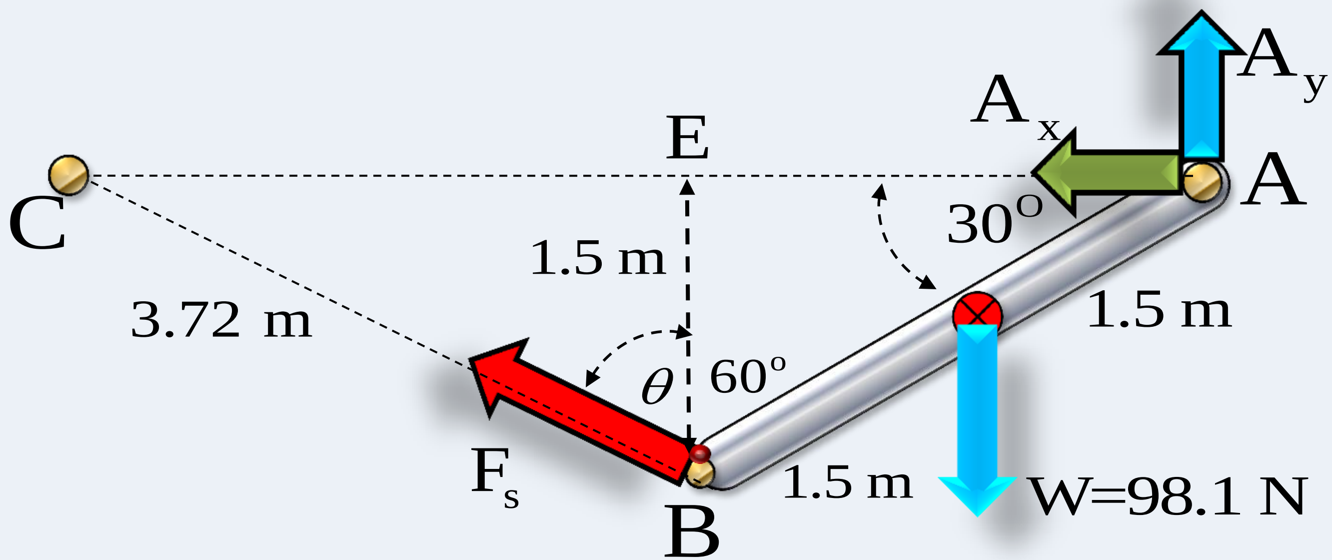
The springs length in equilibrium position

$$BC=\sqrt{6^2+3^2-2(6)(3)\cos 30}=3.72m$$

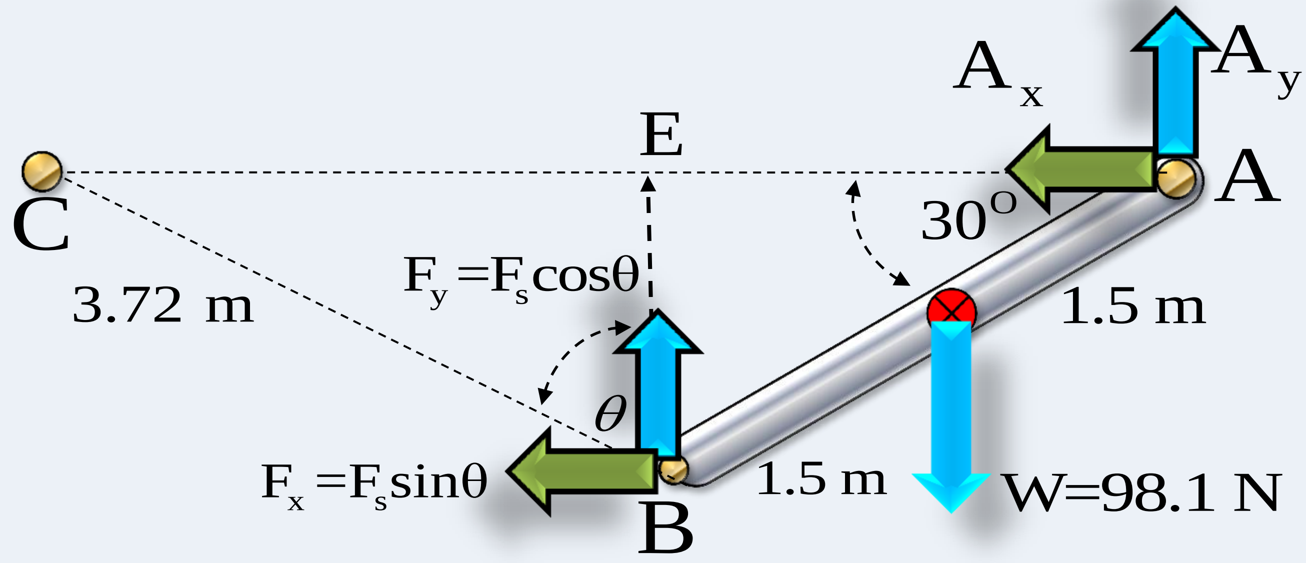
Thus the spring force becomes

$$F_s=k(BC-L_o)$$

$$F_s=k(3.72-3)=0.72k$$



$$\theta = \cos^{-1}\left(\frac{1.5}{3.72}\right) = 66.22^{o}$$



$$\theta = 66.22^o$$

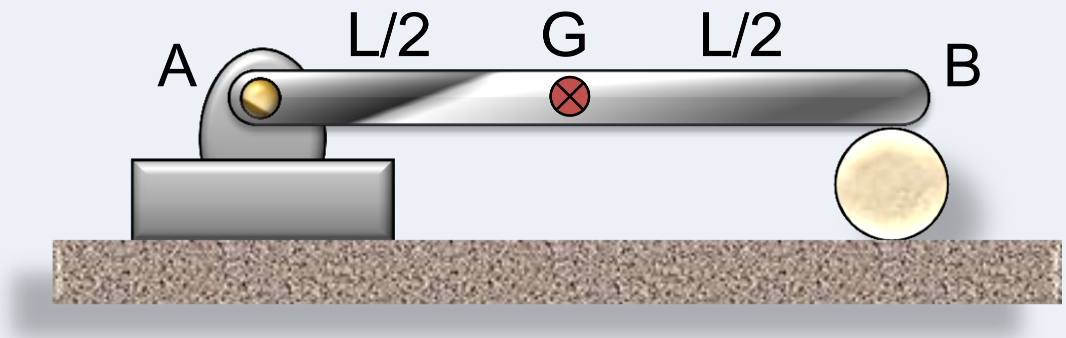
LEFT TO THE STUDENTS

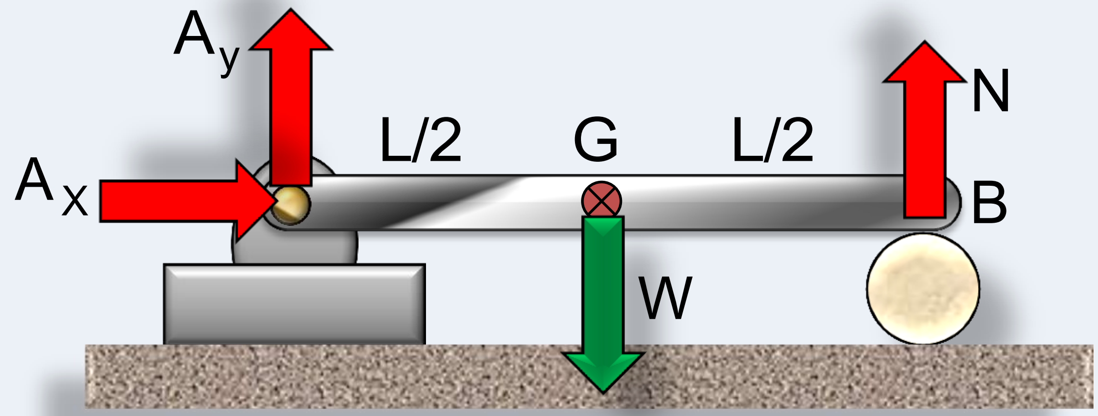
Write down the equilibrium equations
Solve the equations and get the results

IMPROPER EQUILIBRIUM

THE CASE OF A
STATICALLY DETERMINATE SYSTEM

Let us consider the equilibrium of bar AB, weight W and length L





1. Number of **REACTIONS** exerted by the supports are **THREE**

2. The body is in **EQUILIBRIUM** under the given **LOAD (weight).**

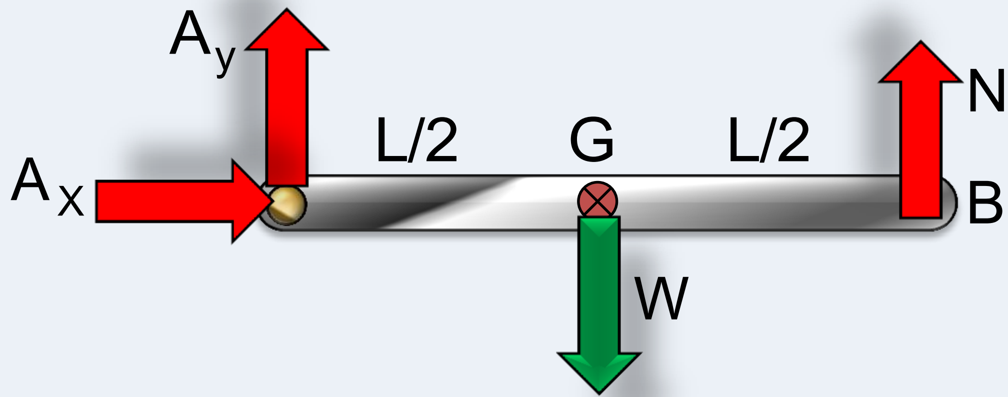
3. The three equilibrium equations are **SATISFIED**.

$$R_x = A_x = 0$$

$$R_y = A_y + N - W = 0$$

$$M_A = N(L) - W\left(\frac{L}{2}\right) = 0$$

4. All three **REACTIONS** are **DETERMINED**



We say that this is a statically determinate system, corresponding to supports that properly constrain the system.

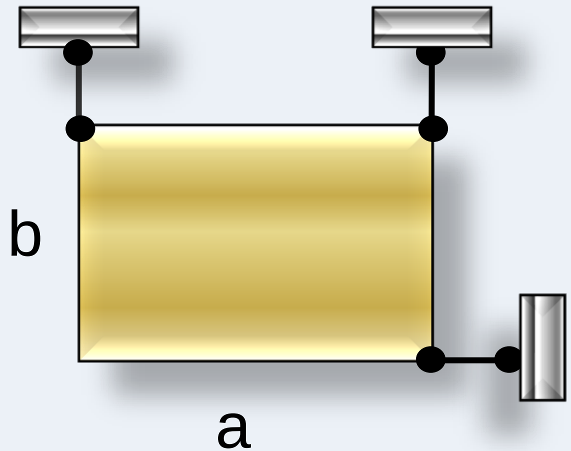
Notice that

a necessary condition for equilibrium :
the three equilibrium equations must be satisfied,

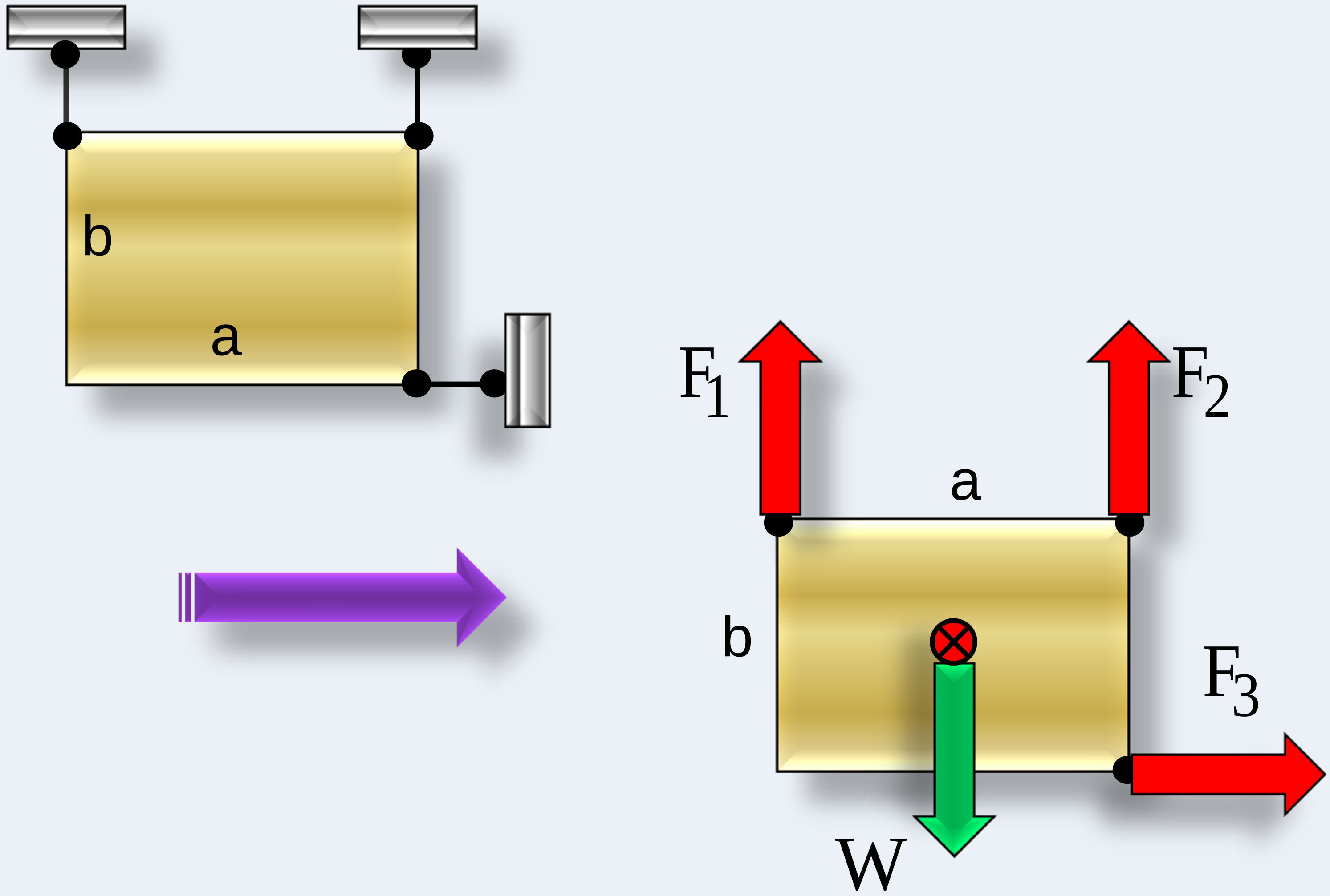
a sufficient condition for equilibrium :
the reaction forces must not constitute a parallel or a concurrent force system.

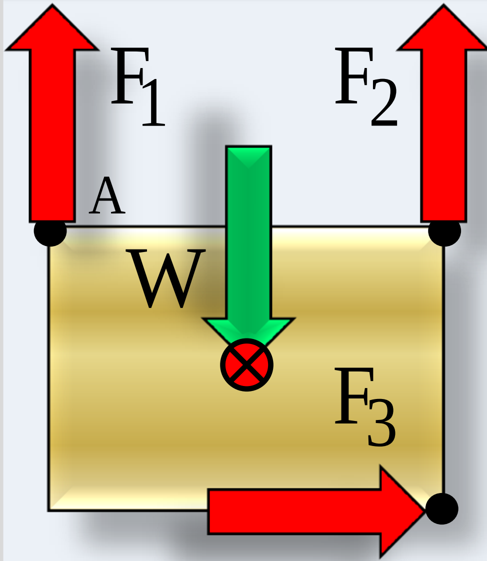
EXAMPLE

The plate is of weight W and held in a vertical plane as shown. All connections consist of smooth pins, rollers, or short links. In each case determine whether (a) the plate is completely, partially, or improperly constrained, (b) the reactions are statically determinate or indeterminate, (c) the equilibrium of the plate is maintained in the position shown.



SOLUTION





1. The REACTIONS exerted by the supports **are THREE**
2. The reactions don't form either a concurrent force system or a parallel force system.
3. Condition for static equilibrium

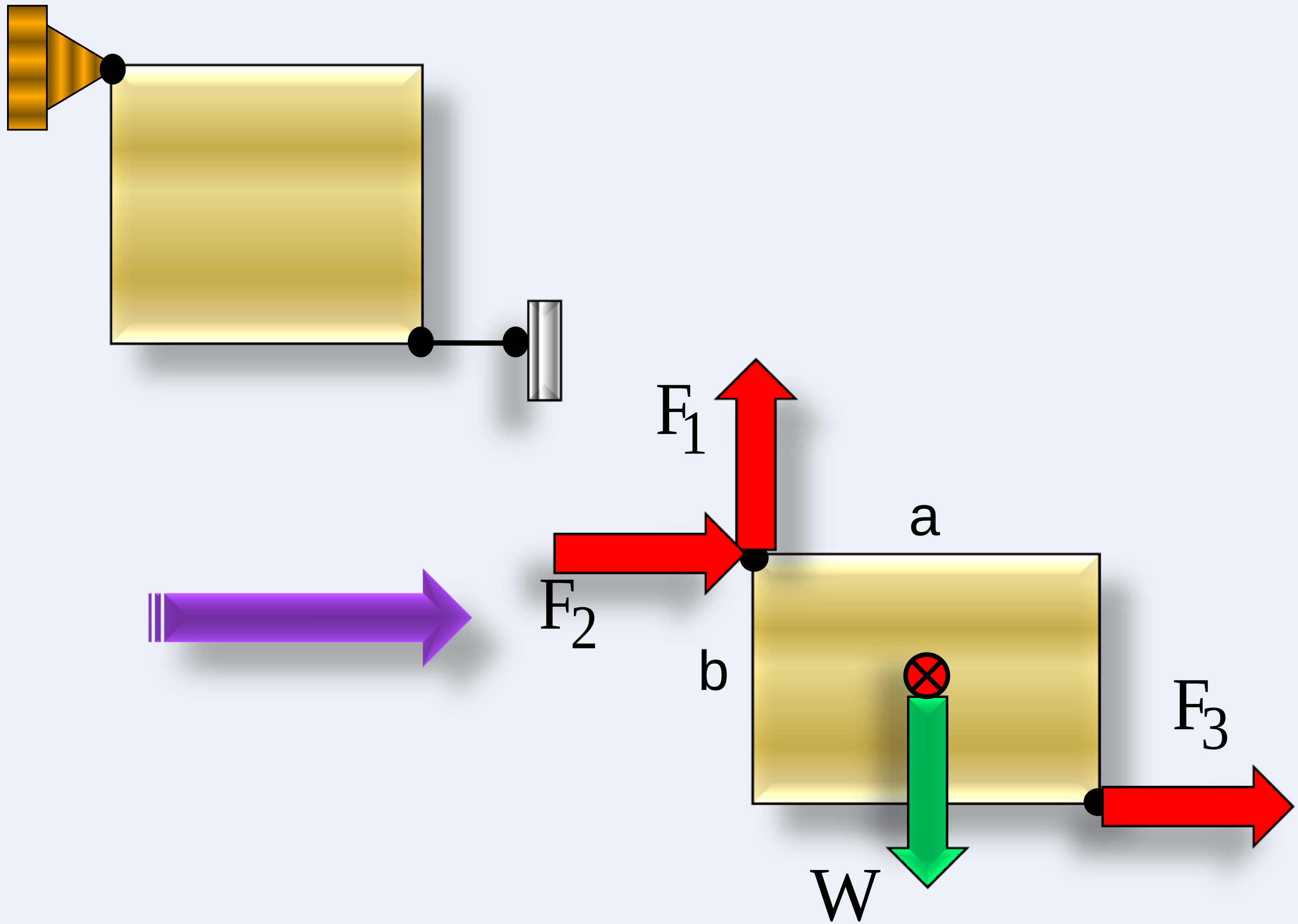
$$R_x = F_3 = 0 \qquad R_y = F_1 + F_2 - W = 0$$

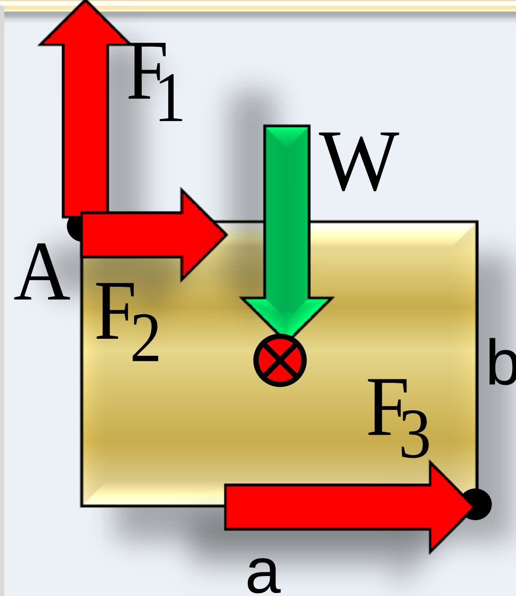
$$M_A = F_2(a) + F_3(b) - W\left(\frac{a}{2}\right) = 0$$

4. All three equations are satisfied
5. The body is in EQUILIBRIUM under the given **LOAD**. AND ANY LOAD
6. All three reactions are DETERMINED

THE SYSTEM IS STATICALLY DETERMINATE

Fig. c





1. Number of reactions exerted by the supports equal **three**

2. The reactions don't form a concurrent force system or a parallel force system

3. Condition for static equilibrium

$$R_x = F_2 + F_3 = 0$$

$$R_y = F_1 - W = 0$$

$$M_A = F_3(b) - W\left(\frac{a}{2}\right) = 0$$

4. All three equations of static equilibrium are satisfied

5. The body is in EQUILIBRIUM under the given **LOAD**. AND ANY LOAD

6. All reactions are DETERMINED

THE SYSTEM IS STATICALLY DETERMINATE